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The IAU Resolutions on Astronomical Reference Systems, Time Scales, and Earth Rotation Models

Explanation and Implementation

by

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Introduction

The series of resolutions passed by the International Astronomical Union at its General Assemblies in 1997 and 2000 are the most significant set of international agreements in positional astronomy in several decades and arguably since the Paris conference of 1896. The approval of these resolutions culminated a process — not without controversy — that began with the formation of an inter-commission Working Group on Reference Systems at the 1985 IAU General Assembly in Delhi.

The resolutions came at the end of a remarkable decade for astrometry, geodesy, and dynamical astronomy. That decade witnessed the successes of the Hipparcos satellite and the Hubble Space Telescope (in both cases, after apparently fatal initial problems), the completion of the Global Positioning System, 25-year milestones in the use of very long baseline interferometry (VLBI) and lunar laser ranging (LLR) for astrometric and geodetic measurements, the discovery of Kuiper Belt objects and extra-solar planets, and the impact of comet Shoemaker-Levy 9 onto Jupiter. At the end of the decade, interest in near-Earth asteroids and advances in sensor design were motivating plans for rapid and deep all-sky surveys. Significant advances in theory also took place, facilitated by inexpensive computer power and the Internet. Positional and dynamical astronomy were enriched by a deeper understanding of chaos and resonances in the solar system, advances in the theory of the rotational dynamics of the Earth, and increasingly sophisticated models of how planetary and stellar systems form and evolve. It is not too much of an exaggeration to say that as a result of these and similar developments, the old idea that astrometry is an essential tool of astrophysics was rediscovered. The IAU resolutions thus came at a fortuitous time, providing a solid framework for interpreting the modern high-precision measurements that are revitalizing so many areas of astronomy.

This circular is an attempt to explain these resolutions and provide guidance on their implementation. This publication is the successor to USNO Circular 163 (1982), which had a similar purpose for the IAU resolutions passed in 1976, 1979, and 1982. Both the 1976–1982 resolutions and those of 1997–2000 provide the specification of the fundamental astronomical reference system, the definition of time scales to be used in astronomy, and the designation of conventional models for Earth orientation calculations (involving precession, nutation, and Universal Time). It will certainly not go unnoticed by readers familiar with Circular 163 that the current publication is considerably thicker. This reflects both the increased complexity of the subject matter and the wider audience that is addressed.

Of course, the IAU resolutions of 1997–2000 did not arise in a vacuum. Many people participated in various IAU working groups, colloquia, and symposia in the 1980s and 1990s on these topics, and some important resolutions were in fact passed by the IAU in the early 1990s. Furthermore, any set of international standards dealing with such fundamental matters as space and time must to some extent be based on, and provide continuity with, existing practice. Therefore, many of the new resolutions carry "baggage" from the past, and there is always the question of how much of this history (some of it quite convoluted) is important for those who simply wish to implement the latest recommendations. Material in this circular generally avoids detailed history in an effort to present the most succinct and least confusing picture possible. However, many readers will be involved with modifying existing software systems, and some mention of previous practice is necessary simply to indicate what needs to be changed. A limited amount of background material also sometimes aids in understanding and provides a context for the new recommendations. The reader should be aware that the presentation of such material is selective and no attempt at historical completeness is attempted.

It must be emphasized that the resolutions described here affect astronomical quantities only at the level of some tens of milliarcseconds or less at the present epoch. And, despite misinformation to the contrary, familiar concepts such as the equinox and sidereal time have not been discarded. The largest systematic change is due to the new rate of precession, which is 0.3 arcsecond per century less than the previous (1976) rate; the change affects some types of astronomical coordinates and sidereal time. Astronomical software applications that work acceptably well now at the arcsecond or 0.1-arcsecond level (which would include most telescope control systems) will continue to work at that level, even when used with new sources of reference data, such as the Hipparcos, Tycho-2, or 2MASS star catalogs or the VCS3 radio source catalog. Applications that are independent of the rotation of the Earth, such as those for differential (small-field) astrometry, are largely unaffected. For these kinds of systems, changes to computer code that implement the new resolutions are recommended as a long-term goal, to maintain standardization of algorithms throughout the astronomical community, but are not an immediate practical necessity. (Perhaps some readers will stop right here and file this circular on the shelf!)

Overview of the Resolutions

The IAU resolutions described in this circular cover a range of fundamental topics in positional astronomy:

- **Relativity** Resolutions passed in 2000 provide the relativistic metric tensors for reference systems with origins at the solar system barycenter and the geocenter, and the transformation between the two systems. While these are mostly of use to theorists — for example, in the formulation of accurate models of observations — they provide a proper relativistic framework for current and future developments in precise astrometry, geodesy, and dynamical astronomy. (See Chapter 1.)
- **Time Scales** Resolutions passed in 1991 and 2000 provide the definitions of various kinds of astronomical time and the relationships between them. Included are time scales based on the Système International (SI) second ("atomic" time scales) as well as those based on the rotation of the Earth. (See Chapter 2.)
- **The Fundamental Astronomical Reference System** A resolution passed in 1997 established the International Celestial Reference System (ICRS), a high precision coordinate system with its origin at the solar system barycenter and "space fixed" (kinematically nonrotating) axes. The resolution included the specification of two sets of benchmark objects and their coordinates, one for radio observations (VLBI-measured positions of pointlike extragalactic sources) and one for optical observations (Hipparcos-measured positions of stars). These two sets of reference objects provide the practical implementation of the system and allow new observations to be related to it. (See Chapter 3.)

- **Precession and Nutation** Resolutions passed in 2000 provided a new precise definition of the celestial pole and endorsed a specific theoretical development for computing its instantaneous motion. The celestial pole to which these developments refer is called the Celestial Intermediate Pole (CIP); the instantaneous equatorial plane is orthogonal to the CIP. There are now new precise algorithms for computing the pole's position on the celestial sphere at any time, in the form of new expressions for precession and nutation. (See Chapter 5.)
- **Earth Rotation** A resolution passed in 2000 establishes new reference points, one on the celestial sphere and one on the surface of the Earth, for the measurement of the rotation of the Earth about its axis. The new points are called, respectively, the Celestial Intermediate Origin (CIO) and the Terrestrial Intermediate Origin (TIO). Both lie in the instantaneous equatorial plane. The rotation of the Earth is simply the geocentric angle, θ, between these two points, a linear function of Universal Time (UT1). The CIO is analogous to the equinox, the reference point on the celestial sphere for sidereal time. Unlike the equinox, however, the CIO has no motion along the instantaneous equator, and unlike sidereal time, θ is not "contaminated" by precession or nutation. The new CIO-TIO-based Earth rotation paradigm thus allows a clean separation of Earth rotation, precession, and nutation in the transformation between terrestrial and celestial reference systems. (See Chapter 6.)

This circular also includes a brief description of the de facto standard solar system model, produced and distributed by the Jet Propulsion Laboratory (see Chapter 4). This model, labeled DE405/LE405, provides the positions and velocities of the nine major planets and the Moon with respect to the solar system barycenter for any date and time between 1600 and 2200. The positions and velocities are given in rectangular coördinates, referred to the ICRS axes. This ephemeris is not the subject of any IAU resolutions but has become widely adopted internationally; for example, it is the basis for the tabulations in *The Astronomical Almanac* and it underlies some of the other algorithms presented in this circular.

The 1997 and 2000 IAU resolutions form an interrelated and coherent set of standards for positional astronomy. For example, the definitions of the SI-based time scales rely on the relativity resolutions, and the position of the Celestial Intermediate Pole and the Celestial Intermediate Origin can only be properly computed using the new precession and nutation expressions. Many other links between the resolutions exist. In fact, attempting to apply the resolutions selectively can lead to quite incorrect (or impossible to interpret) results. This circular is meant to provide an explanatory and computational framework for a holistic approach to implementing these resolutions in various astronomical applications. The author hopes that what is presented here does justice to the efforts of the many people who worked very hard over the last decade to take some important scientific ideas and work out their practical implications for positional astronomy, to the benefit of the entire scientific community.

About this Circular

The chapters in this circular reflect the six main subject areas described above. Each of the chapters contains a list of the relevant IAU resolutions, a summary of the recommendations, an explanatory narrative, and, in most chapters, a collection of formulas used in implementing the recommendations. The references for all chapters are collected in one list at the end of the circular (p. 82 ff.). The reference list is in the usual style for astronomy journal articles. At the end of the references, a list of Uniform Resource Locators (URLs) is given (p. 87) for documents and data that are available on the World Wide Web. These URLs are numbered and they are referred to in the text by number — for example, a PDF version of this circular can be found at URL 1.

It is assumed that readers have a basic knowledge of positional astronomy; that the terms right ascension, declination, sidereal time, precesssion, nutation, equinox, ecliptic, and ephemeris are familiar. Some experience in computing some type of positional astronomy data is useful, because the ultimate purpose of the circular is to enable such computations to be carried out in accordance with the 1997 and 2000 IAU resolutions. The explanatory narratives deal primarily with new or unfamiliar concepts introduced by the resolutions — concepts that would not generally be described in most introductory textbooks on positional astronomy. This circular is not a substitute for such textbooks.

IAU resolutions are referred to in the text in the form "res. N of year", for example, "res. B1.2 of 2000". The year refers to the year of the IAU General Assembly that passed the resolution. The proceedings of each General Assembly, including the text of the resolutions, are usually published the following year. The References section of this circular lists the various proceedings volumes under "IAU". An online reference for the text of IAU resolutions (beginning with those passed at the 1994 General Assembly) is the IAU Information Bulletin (IB) series, at URL 2. Resolutions are printed in the January IB following a General Assembly, i.e., IB numbers 74, 81, 88, 94, etc. This circular contains two appendices containing the complete text of the resolutions passed by the 1997 and 2000 General Assemblies, which are the main focus of attention here.

Errata in this circular and updates to it are provided at URL 1.

Other Resources

An increasing number of publications, data, and software related to the recent IAU resolutions are becoming available.

A major online resource for implementing the IAU resolutions involving Earth rotation and time (Chapters 2, 5, and 6 here) is the document of conventions used by the International Earth Rotation and Reference Systems Service (IERS): IERS Conventions (2003), IERS Technical Note No. 32, edited by D. D. McCarthy and G. Petit. It is available in printed form from the IERS and also on the web at URL 3. The online document contains links to Fortran subroutines that implement the recommended models. The document also contains algorithms specific to geodetic applications, such as tidal and geopotential models, that have not been the subject of IAU action and are not discussed in this circular. The IERS also maintains an online list of FAQs on the IAU resolutions (URL 4).

The IAU Working Group on Nomenclature for Fundamental Astronomy (2003–2006) has a website (URL 6) with many helpful documents, including a list of definitions (some of which are used in this circular) and other educational material.

In addition to the IERS software, two other packages of computer subroutines are available for implementing the IAU resolutions: the Standards of Fundamental Astronomy (SOFA), at URL 7, and the Naval Observatory Vector Astrometry Subroutines (NOVAS), at URL 8. SOFA is a collection of routines managed by an international panel, the SOFA Reviewing Board, that works under the auspices of IAU Division 1 and is chaired by P. Wallace. The board has adopted a set of Fortran coding standards for algorithm implementations (C versions are contemplated for the future) and is soliciting code from the astrometric and geodetic communities that implements IAU models. Subroutines are adapted to the coding standards and validated for accuracy before being added to the SOFA collection. NOVAS is an integrated package of subroutines, available in Fortran and

C, for the computation of a wide variety of common astrometric quantities and transformations. NOVAS dates back to the 1970s but has been continually updated to adhere to subsequent IAU resolutions.

The Astronomical Almanac, beginning with the 2006 edition, is also a resource for implementing the IAU resolutions. Not only does it list various algorithms arising from or consistent with the resolutions, but its tabular data serve as numerical checks for independent developments. Both SOFA and NOVAS subroutines are used in preparing the tabulations in *The Astronomical Almanac*, and various checks have been made to ensure the consistency of the output of the two software packages.

Acknowledgments

Many people have contributed in some way to this circular. I have had many interesting and enlightening discussions with James Hilton, Victor Slabinski, and Patrick Wallace. I wish to thank Bill Tangren for setting me straight on some of the basic concepts of general relativity, and Sergei Klioner and Sergei Kopeikin for patiently trying to explain to me the relativistic aspects of the 2000 IAU resolutions. Sergei Klioner provided some text that I have used verbatim. I hope that the final product properly represents their input. The e-mail discussions within the IAU Working Group on Nomenclature for Fundamental Astronomy have also been quite valuable and have contributed to what is presented here. The many scientific papers written by Nicole Capitaine and her collaborators have been essential references. Victor Slabinski and Dennis McCarthy of the U.S. Naval Observatory, Ken Seidelmann of the University of Virginia, Catherine Hohenkerk of Her Majesty's Nautical Almanac Office, and Myles Standish of the Jet Propulsion Laboratory carefully reviewed drafts of this circular and made many substantive suggestions for improvement. The remaining defects in the circular are, of course, my sole responsibility.

— George Kaplan

A Few Words about Constants

This circular does not contain a list of adopted fundamental astronomical constants, because the IAU is no longer maintaining such a list. The last set of officially adopted constant values was the IAU (1976) System of Astronomical Constants. That list is almost entirely obsolete. For a while, an IAU working group maintained a list of "best estimates" of various constant values, but the IAU General Assembly of 2003 did not renew that mandate. It can be argued that a list of fundamental astronomical constants is no longer possible, given the complexity of the models now used and the many free parameters that must be adjusted in each model to fit observations. That is, there are more constants now to consider, and their values are theory dependent. In many cases, it would be incorrect to attempt to use a constant value, obtained from the fit of one theory to observations, with another theory.

We are left with three *defining constants* with IAU-sanctioned values that are intended to be fixed:

1. The Gaussian gravitational constant: $k = 0.01720209895$. The dimensions of k^2 are $\mathrm{AU}^3\mathrm{M}_\odot^{-1}\mathrm{d}^{-2}$ where AU is the astronomical unit, $\mathrm{M}_\odot$ is the solar mass, and d is the day of 86400 seconds.
2. The speed of light: $c = 299\,792\,458\ \mathrm{m\,s}^{-1}$.
3. The fractional difference in rate between the time scales TT and TCG: $\mathrm{L}_G = 6.969290134\times10^{-10}$. Specifically, the derivative $d\mathrm{TT}/d\mathrm{TCG} = 1 - \mathrm{L}_G$. (See Chapter 3.)

The IERS Conventions (2003) includes a list of constants as its Table 1.1. Several useful ones from this list that are not highly theory dependent (for astronomical use, at least) are:

1. Equatorial radius of the Earth: $a_E = 6\,378\,136.6$ m.
2. Flattening factor of the Earth: $f = 1/298.25642$.
3. Dynamical form factor of the Earth: $J_2 = 1.0826359\times10^{-3}$.
4. Nominal mean angular velocity of Earth rotation: $\omega = 7.292115\times10^{-5}\ \mathrm{rad\,s}^{-1}$.
5. Constant of gravitation: $G = 6.673\times10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$ (CODATA 2002 recommended value: $6.6742\times10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$).

The first four values above were recommended by Special Commission 3 of the International Association of Geodesy; the first three are so-called "zero tide" values. (The need to introduce the concept of "zero tide" values indicates how theory creeps into even such basic constants as the radius of the Earth as the precision of measurement increases. See section 1.1 of the IERS

Conventions (2003).) Planetary masses, the length of the astronomical unit, and related constants used in or obtained from the Jet Propulsion Laboratory DE405/LE405 ephemeris are listed with its description in Chapter 4. The rate of general precession in longitude (the "constant of precession") is given in Chapter 5 on the precession and nutation theories.

The World Geodetic System 1984 (WGS 84), which is the basis for coordinates obtained from GPS, uses an Earth ellipsoid with $a_E = 6378137$ m and $f = 1/298.257223563$.

Some astronomical "constants" (along with reference data such as star positions) actually represent quantities that slowly vary, and the values given must therefore be associated with a specific epoch. That epoch is now almost always 2000 January 1, 12^{h} (JD 2451545.0), which can be expressed in any of the usual time scales. If, however, that epoch is considered an event at the geocenter and given in the TT time scale, the epoch is designated J2000.0. See Chapter 2.

Abbreviations and Symbols Frequently Used

and index to most relevant sections

α	right ascension	3.2
δ	declination	3.2
$\Delta\psi$	nutation in [ecliptic] longitude (usually expressed in arcseconds)	5.3, 5.4.2
$\Delta\epsilon$	nutation in obliquity (usually expressed in arcseconds)	5.3, 5.4.2
ϵ	mean obliquity of date	5.4.1
ϵ'	true obliquity of date ($= \epsilon + \Delta\epsilon$)	5.4.2
ϵ_0	mean obliquity of J2000.0	5.3, 5.4.1
θ	Earth Rotation Angle	2.3, 6.2
μas	microarcecond ($= 10^{-6}$ arcsecond $\approx 4.8\times10^{-12}$ radian)	
σ	a non-rotating origin or, specifically, the CIO	6.2
$\boldsymbol{\sigma}$	unit vector toward a non-rotating origin or, specifically, the CIO	6.5.1
Υ	the equinox	3.2, 6.2
$\boldsymbol{\Upsilon}$	unit vector toward the equinox	6.5.1
as *or* $''$	arcsecond ($= 1/3600$ degree $\approx 4.8\times10^{-6}$ radian)	
AU	astronomical unit(s)	
B	frame bias matrix	3.5
BCRS	Barycentric[1] Celestial Reference System	1.2
BIPM	Bureau International des Poids et Mesures	
C	matrix for transformation from GCRS to E_σ	5.4.3, 6.5.3
cen	century, specifically, the Julian century of 36525 days of 86400 seconds	
CIO	Celestial Intermediate Origin[2,3]	6.2, 6.5.1
CIP	Celestial Intermediate Pole	5.2, 6.5.1
CIRS	(See E_σ)	
E_Υ	instantaneous (true) equator and equinox of date	6.4, 6.5.3
E_σ	Celestial Intermediate Reference System (CIRS)	6.4, 6.5.3
E_ϖ	Terrestrial Intermediate Reference System (TIRS)	6.4, 6.5.3
$\mathcal{E}_\Upsilon$	equation of the equinoxes	2.3, 2.6.2
$\mathcal{E}_o$	equation of the origins	6.5.1.1, 6.5.4
ESA	European Space Agency	
FKn	n^{th} Fundamental Catalog (Astronomisches Rechen-Institut, Heidelberg)	3.3
GAST	Greenwich apparent sidereal time	2.3, 2.6.2
GCRS	Geocentric Celestial Reference System	1.2

GMST	Greenwich mean sidereal time	2.3, 2.6.2
GPS	Global Positioning System	
HCRF	Hipparcos Celestial Reference Frame	3.1, 3.4
IAG	International Association of Geodesy	
IAU	International Astronomical Union	
ICRF	International Celestial Reference Frame	3.1, 3.4
ICRS	International Celestial Reference System	3.1, 3.4
IERS	International Earth Rotation and Reference System Service	
ITRF	International Terrestrial Reference Frame	6.4
ITRS	International Terrestrial Reference System	6.4, 6.5.2
IUGG	International Union of Geodesy and Geophysics	
J2000.0	the epoch 2000 January 1, 12^{h} TT (JD 2451545.0 TT) at the geocenter ("J2000.0 system" is shorthand for the celestial reference system defined by the mean dynamical equator and equinox of J2000.0.)	2.2 3.2
JD	Julian date (time scale used should be specified)	
JPL	Jet Propulsion Laboratory	
mas	milliarcsecond ($= 10^{-3}$ arcsecond $\approx 4.8\times10^{-9}$ radian)	
$\mathbf{N}$	nutation matrix (for transformation from mean to true system of date)	5.4, 5.4.2
$\mathbf{n}$	unit vector toward the CIP (celestial pole)	5.4,6.5.1
NOVAS	Naval Observatory Vector Astrometry Subroutines (software)	
$\mathbf{P}$	precession matrix (for transformation from J2000.0 system to mean system of date)	5.4, 5.4.2

$\mathbf{R}_1(\phi)$ rotation matrix to transform column 3-vectors from one cartesian coordinate system to another. Final system is formed by rotating original system about its own x-axis by angle ϕ (counterclockwise as viewed from the +x direction):

$$\mathbf{R}_1(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix}$$

$\mathbf{R}_2(\phi)$ rotation matrix to transform column 3-vectors from one cartesian coordinate system to another. Final system is formed by rotating original system about its own y-axis by angle ϕ (counterclockwise as viewed from the +y direction):

$$\mathbf{R}_2(\phi) = \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix}$$

$\mathbf{R}_3(\phi)$ rotation matrix to transform column 3-vectors from one cartesian coordinate system to another. Final system is formed by rotating original system about its own z-axis by angle ϕ (counterclockwise as viewed from the +z direction):

$$\mathbf{R}_3(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

s	CIO locator: the difference between two arcs on the celestial sphere, providing the direction toward the CIO	6.5.1.3
SI	Système International d'Unités (International System of Units)	
SOFA	Standards of Fundamental Astronomy (software)	
T	unless otherwise specified, time in Julian centuries (36525 days of 86400 seconds) from JD 2451545.0 (2000 Jan 1.5) (The time scale used should be specified, otherwise TT is understood.)	
$\mathrm{T_{eph}}$	time argument of JPL planetary and lunar ephemerides	2.2, 4.2
TAI	International Atomic Time	2.2
TCB	Barycentric[1] Coordinate Time	1.2, 2.2
TCG	Geocentric Coordinate Time	1.2, 2.2
TDB	Barycentric[1] Dynamical Time	2.2
TIO	Terrestrial Intermediate Origin[2]	6.2, 6.5.2
TIRS	(See E_ϖ)	
TT	Terrestrial Time	2.2
UCAC	USNO CCD Astrographic Catalog	3.4.5
USNO	U.S. Naval Observatory	
UT1	Universal Time (affected by variations in length of day)	2.3, 2.6.2
UTC	Coordinated Universal Time (an atomic time scale)	2.4
VLBI	very long baseline [radio] interferometry	
$\mathbf{W}$	"wobble" (polar motion) matrix (for transformation from ITRS to E_ϖ)	6.5.2
WGS 84	World Geodetic System 1984	6.4
X Y Z	components of $\mathbf{n}_{\mathrm{GCRS}}$, unit vector toward the CIP with respect to the GCRS	5.4, 6.5.1
x_p y_p	standard polar motion parameters, defining location of the CIP in the ITRS	6.5.2

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[1] "Barycentric" always refers to the solar system barycenter, the center of mass of all bodies in the solar system.

[2] The fundamental reference points referred to here as the Celestial Intermediate Origin (CIO) and the Terrestrial Intermediate Origin (TIO) were called, respectively, the Celestial Ephemeris Origin (CEO) and the Terrestrial Ephemeris Origin (TEO) in the IAU resolutions of 2000. The IAU Working Group on Nomenclature for Fundamental Astronomy (URL 6) has recommended the change of nomenclature with no change in the definitions. The new terminology is already in use in *The Astronomical Almanac* and in IERS documents, and will undoubtedly be adopted by the IAU General Assembly in 2006. It is used throughout this circular, except in the verbatim text of the IAU resolutions.

[3] The abbreviation CIO was used throughout much of the 20th century to designate the Conventional International Origin, the reference point for the measurement of polar motion.

Chapter 1

Relativity

Relevant IAU resolutions: A4.I, A4.II, A4.III, A4.IV of 1991; B1.3, B1.4, B1.5 of 2000

Summary In 2000, the IAU defined a system of space-time coordinates for (1) the solar system, and (2) the Earth, within the framework of General Relativity, by specifying the form of the metric tensors for each and the 4-dimensional space-time transformation between them. The former is called the Barycentric Celestial Reference System (BCRS) and the latter is called the Geocentric Celestial Reference System (GCRS). The BCRS is the system appropriate for the basic ephemerides of solar system objects and astrometric reference data on galactic and extragalactic objects. The GCRS is the system appropriate for describing the rotation of the Earth, the orbits of Earth satellites, and geodetic quantities such as instrument locations and baselines. The analysis of precise observations inevitably involves quantities expressed in both systems and the transformations between them.

1.1 Background

Although the theory of relativity has been with us for a century (Einstein's first papers on special relativity were published in 1905), it has only been within the last few decades that it has become a routine consideration in positional astronomy. The reason is simply that the observational effects of both special and general relativity are small. In the solar system, deviations from Newtonian physics did not need to be taken into account — except for the advance of the perihelion of Mercury — until the advent of highly precise "space techniques" in the 1960s and 1970s: radar ranging, spacecraft ranging, very long baseline interferometry (VLBI), pulsar timing, and lunar laser ranging (LLR). More recently, even optical astrometry has joined the list, with wide-angle satellite measurements (Hipparcos) at the milliarcsecond level. Currently, the effects of relativity are often treated as small corrections added to basically Newtonian developments. But it has become evident that the next generation of instrumentation and theory will require a more comprehensive approach, one that encompasses definitions of such basic concepts as coordinate systems, time scales, and units of measurement in a relativistically consistent way. It may remain the case that, for many applications, relativistic effects can either be ignored or handled as second-order corrections to Newtonian formulas. However, even in such simple cases, the establishment of a self-consistent relativistic framework has benefits — it at least allows the physical assumptions and the errors involved to be more clearly understood.

In 1991, the IAU made a series of recommendations concerning how the theory of relativity could best be incorporated into positional astronomy. These recommendations and their implications were studied by several working groups in the 1990s and some deficiencies were noted. As a result, a series of new recommendations was proposed and discussed at IAU Colloquium 180 (Johnston et al. 2000). The new recommendations were passed by the IAU General Assembly in 2000. It is these recommendations that are described briefly in this chapter.

In special relativity, the Newtonian idea of absolute time in all inertial reference systems is replaced by the concept that time runs differently in different inertial systems, in such a way that the speed of light has the same measured value in all of them. In both Newtonian physics and special relativity, inertial reference systems are preferred: physical laws are simple when written in terms of inertial coordinates. In general relativity, however, time (and even space-time) is influenced not only by velocity but also by gravitational fields, and there are no preferred reference systems. One can use, in principle, any reference system to model physical processes. For an infinitely small space-time region around an observer (considered to be a massless point), one can introduce so-called locally inertial reference systems where, according to the Einstein's equivalence principle, all physical laws have the same form as in an inertial reference system in special relativity. Such locally inertial reference systems are used to describe observations taken by the point-like observer. In general-relativistic reference systems of finite spatial extent, the geometry of space-time is defined by a *metric tensor*, a 4×4 matrix of mathematical expressions, that serves as an operator on two 4-vectors. In its simplest application, the metric tensor directly yields the generalized (4-dimensional) distance between two neighboring space-time events. The metric tensor effectively determines the equations through which physics is described in the reference system.

Time in general relativity can be understood as follows. As a particle moves through space-time, each point (a space-time event) on the path that it follows can be characterized by a set of four numbers. These four numbers are the values of the four coordinates in four-dimensional space-time for a given coordinate system. For the same path in a different coordinate system, the numbers will, in general, be different. *Proper time* is simply the time kept by a clock co-moving with the particle, in whatever trajectory and gravity field it finds itself. Proper time is always measurable if a clock is available that can travel with the particle. *Coordinate time* is one of the four independent variables used to characterize a space-time event. Coordinate time is *not* measurable. The coordinate time of a reference system is the independent argument of the equations of motion of bodies in that reference system. The IAU resolutions on relativity passed in 2000 are concerned with two coordinate frames, one barycentric and one geocentric, and the coordinate times used in each one.

1.2 The BCRS and the GCRS

In res. B1.3 of 2000, the IAU defined two coordinate frames for use in astronomy, one with its origin at the solar system barycenter and one with its origin at the geocenter. In current astronomical usage these are referred to as *reference systems.* (The astronomical distinction between *reference systems* and *reference frames* is discussed in Chapter 3.) The two systems are the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS). Harmonic coordinates are recommended for both systems (i.e., the harmonic gauge is used). The resolution provides the specific forms of the metric tensors for the two coordinate systems and the 4-dimensional transformation between them. (The latter would reduce to a Lorentz transformation for a fictitious Earth moving with constant velocity in the absence of gravitational fields.) The general forms of the gravitational potentials, which appear in the metric tensors, are also presented. In res. B1.4, specific expansions of the Earth's gravitational potential in the GCRS are recommended. In res. B1.5, the relationship between the coordinate time scales for the two reference systems, Barycentric Coordinate Time (TCB), and Geocentric Coordinate Time (TCG), is given. Each of the resolutions is mathematically detailed, and the formulas may be found in the text of the resolutions at the end of this circular. For interested readers, the paper titled "The IAU 2000 Resolutions for Astrometry, Celestial Mechanics, and Metrology in the Relativistic Framework: Explanatory Supplement" (Soffel et al. 2003), is highly recommended as a narrative on the background, meaning, and application of the relativity resolutions. Here we will make only very general comments on the BCRS and GCRS, although the time scales TCB and TCG are described in a bit more detail in Chapter 2.

The BCRS is a "global" reference system in which the positions and motions of bodies outside the immediate environment of the Earth are to be expressed. It is the reference system appropriate for the solution of the equations of motion of solar system bodies (that is, the development of solar system ephemerides) and within which the positions and motions of galactic and extragalactic objects are most simply expressed. It is the system to be used for most positional-astronomy reference data, e.g., star catalogs. The GCRS is a "local" reference system for Earth-based measurements and the solution of the equations of motion of bodies in the near-Earth environment, e.g., artificial satellites. The time-varying position of the Earth's celestial pole is defined within the GCRS (res. B1.7 of 2000). Precise astronomical observations involve both systems: the instrumental coordinates, boresights, baselines, etc., may be expressed in the GCRS, but in general we want the astronomical results expressed in the BCRS where they are easier to interpret. Thus it is unavoidable that data analysis procedures for precise techniques will involve both GCRS and BCRS quantities and the transformation between them. For example, the basic equation for VLBI delay (the time difference between wavefront arrivals at two antennas) explicitly involves vectors expressed in both systems — antenna-antenna baselines are given in the GCRS, while solar system coordinates and velocities and quasar directions are expressed in the BCRS. Various relativistic factors connect the two kinds of vectors.

In the 2000 resolutions, the coordinate axes of the two reference systems do not have a defined orientation. They are described as *kinematically nonrotating*, which means that the axes have no systematic rotation with respect to distant objects in the universe (and specifically the radio sources that make up the ICRF — see Chapter 3). Since the axis directions are not specified, one interpretation of the 2000 resolutions is that the BCRS and GCRS in effect define families of coordinate systems, the members of which differ only in overall orientation. The IAU Working Group on Nomenclature for Fundamental Astronomy has recommended that the directions of the coordinate axes of the BCRS be understood to be those of the International Celestial Reference System (ICRS) described in Chapter 3. And, since the transformation between the BCRS and GCRS is specified in the resolutions, the directions of the GCRS axes are also implicitly defined by this understanding. Here are the definitions of the two systems recommended by the working group:

> **Barycentric Celestial Reference System (BCRS):** A system of barycentric space-time coordinates for the solar system within the framework of General Relativity with metric tensor specified by the IAU 2000 Resolution B1.3. Formally, the metric tensor of the BCRS does not fix the coordinates completely, leaving the final orientation of the spatial axes undefined. However, for all practical applications, unless otherwise stated, the BCRS is assumed to be oriented according to the ICRS axes.

Geocentric Celestial Reference System (GCRS): A system of geocentric space-time coordinates within the framework of General Relativity with metric tensor specified by the IAU 2000 Resolution B1.3. The GCRS is defined such that the transformation between BCRS and GCRS spatial coordinates contains no rotation component, so that GCRS is kinematically non-rotating with respect to BCRS. The equations of motion of, for example, an Earth satellite with respect to the GCRS will contain relativistic Coriolis forces that come mainly from geodesic precession. The spatial orientation of the GCRS is derived from that of the BCRS, that is, unless otherwise stated, by the orientation of the ICRS.

Because, according to the last sentence of the GCRS definition, the orientation of the GCRS is determined by that of the BCRS, and therefore the ICRS, in this circular the GCRS will often be described as the "geocentric ICRS". However, this sentence does not imply that the spatial orientation of the GCRS is the *same* as that of the BCRS (ICRS). The relative orientation of these two systems is embodied in the 4-dimensional transformation given in res. B1.3 of 2000, which, we will see in the next section, is itself embodied in the algorithms used to compute observable quantities from BCRS (ICRS) reference data. From another perspective, the GCRS is just a rotation (or series of rotations) of the international geodetic system (discussed in Chapter 6). The geodetic system rotates with the crust of the Earth, while the GCRS has no systematic rotation relative to extragalactic objects.

The above definition of the GCRS also indicates some of the subtleties involved in defining the spatial orientation of its axes. Without the kinematically non-rotating constraint, the GCRS would have a slow rotation with respect to the BCRS, the largest component of which is called geodesic (or de Sitter-Fokker) precession. This rotation, approximately 1.9 arcseconds per century, would be inherent in the GCRS if its axes had been defined as *dynamically* non-rotating rather than *kinematically* non-rotating. By imposing the latter condition, Coriolis terms must be added (via the inertial parts of the potentials in the metric; see notes to res. B1.3 of 2000) to the equations of motion of bodies expressed in the GCRS. For example, as mentioned above, the motion of the celestial pole is defined within the GCRS, and geodesic precession appears in the precession-nutation theory rather than in the transformation between the GCRS and BCRS. Other barycentric-geocentric transformation terms that affect the equations of motion of bodies in the GCRS because of the axis-orientation constraint are described in Soffel et al. (2003, section 3.3) and Kopeikin & Vlasov (2004, section 6).

1.3 Computing Observables

Ultimately, the goal of these theoretical formulations is to facilitate the accurate computation of the values of observable astrometric quantities (transit times, zenith distances, focal plane coordinates, interferometric delays, etc.) at the time and place of observation, that is, in the *proper reference system* of the observer. There are some subtleties involved because in Newtonian physics and special relativity, observables are directly related to some inertial coordinate, while according to the rules of general relativity, observables must be computed in a coordinate-independent manner.

In any event, to obtain observables, there are a number of calculations that must be performed. These begin with astrometric reference data: a precomputed solar system ephemeris and, if a star is involved, a star catalog with positions and proper motions listed for a specified epoch. The computations account for the space motion of the object (star or planet), parallax (for a star) or light-time (for a planet), gravitational deflection of light, and the aberration of light due to the

Earth's motions. Collectively, these calculations will be referred to in this circular as the algorithms for *proper place*. For Earth-based observing systems, we must also account for precession, nutation, Earth rotation, and polar motion. There are classical expressions for all these effects (except gravitational deflection), and relativity *explicitly* enters the procedure in only a few places, usually as added terms to the classical expressions and in the formulas that link the various time scales used. It has become common, then, to view this ensemble of calculations as being carried out entirely in a single reference system; or, two reference systems, barycentric and geocentric, that have parallel axes and differ only in the origin of coordinates (that is, they are connected by a Galilean transformation). For example, the coordinate system defined by the "equator and equinox of J2000.0", can be thought of as either barycentric or geocentric. The relativistic effects then are interpreted simply as "corrections" to the classical result.

While such a viewpoint may be aesthetically tidy, it breaks down at high levels of accuracy and for some types of observations. Relativity theory leads to a more correct, albeit more subtle, interpretation for the same set of calculations. It is represented by the BCRS-GCRS paradigm wherein some of the quantities are expressed relative to the BCRS and others are relative to the GCRS. The two systems are quite different in a number of ways, as described in the previous section. The situation is easiest to describe if we restrict the discussion to a fictitious observer at the center of the Earth, that is, to observations referred to the geocenter. The transformation between the two systems is not explicit in the normal algorithms, but is embodied in the relativistic terms in the expressions used for aberration or VLBI delay. The distinction between the two systems is most obvious in the formulation for angular variables. There, the algorithms for space motion, parallax, light-time, and gravitational deflection[1] all use vectors expressed in the BCRS (star catalogs and solar system ephemerides are inherently BCRS), while the series of rotations for precession, nutation, Earth rotation, and polar motion (if applied in that order) starts with vectors expressed in the GCRS. In essence, the aberration calculation connects the two systems because it contains the transformation between them: its input is a pair of vectors in the BCRS and its output is a vector in the GCRS. In the VLBI case, aberration does not appear explicitly, but the conventional algorithm for the delay observable incorporates vectors expressed in both systems, with appropriate conversion factors obtained from the BCRS-GCRS transformation.[2]

For an observer on or near the Earth's surface the calculations have to include the position and velocity of the observer relative to the geocenter. These are naturally expressed in the GCRS but for some of the calculations (parallax, light-time, light deflection, and aberration) they must be added to the position and velocity of the geocenter relative to the solar system barycenter, which are expressed in the BCRS. Thus another GCRS-BCRS transformation is indicated, although the velocity is sufficiently small that a Galilean transformation (simple vector addition) suffices for current observational accuracy (Klioner 2003). Correct use of the resulting vectors results in the values of the observables expressed, not in the GCRS, but in the proper reference system of the observer.

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[1] In the case of the observer at the geocenter, we neglect the gravity field of the Earth itself in computing gravitational deflection.

[2] Part of the expression for VLBI delay, in the time domain, accounts for what would be called aberration in the angular domain; it is possible to compute aberration from the VLBI delay algorithm. See Kaplan (1998).

1.4 Concluding Remarks

The 2000 IAU resolutions on relativity define a framework for future dynamical developments within the context of general relativity. For example, Klioner (2003) has described how to use the framework to compute the directions of stars as they would be seen by a precise observing system in Earth orbit. However, there is much unfinished business. The apparently familiar concept of the ecliptic plane has not yet been defined in the context of relativity resolutions. A consistent relativistic theory of Earth rotation is still some years away; the algorithms described in Chapter 5 are not such a theory, although they contain all the main relativistic effects and are quite adequate for the current observational precision.

A local reference system similar to the GCRS can be easily constructed for any body of an N-body system in exactly the same way as the GCRS, simply by changing the notation so that the subscript E denotes a body other than the Earth. In particular, a celenocentric reference system for the Moon plays an important role in lunar laser ranging.

It is also worth noting that the 2000 resolutions do not describe the proper reference system of the observer — the local, or topocentric, system in which most measurements are actually taken. (VLBI observations are unique in that they exist only after data from various individual antennas are combined; therefore they are referred to the GCRS *ab initio*.) A kinematically non-rotating version of the proper reference system of the observer is just a simplified version of the GCRS: x^i_E should be understood to be the BCRS position of the observer (v^i_E and a^i_E are then the observer's velocity and acceleration) and one should neglect the internal potentials. See Klioner & Voinov (1993); Kopeikin (1991); Kopeikin & Vlasov (2004); Klioner (2004).

One final point: the 2000 IAU resolutions as adopted apply specifically to Einstein's theory of gravity, i.e., the general theory of relativity. The Parameterized Post-Newtonian (PPN) formalism (see, e.g., Will & Nordtvedt (1972)) is more general, and the 2000 resolutions have been discussed in the PPN context by Klioner & Soffel (2000) and Kopeikin & Vlasov (2004). In the 2000 resolutions, it is assumed that the PPN parameters β and γ are both 1.

Chapter 2

Time Scales

Relevant IAU resolutions: A4.III, A4.IV, A4.V, A4.VI of 1991; C7 of 1994; B1.3, B1.5, B1.7, B1.8, B1.9, and B2 of 2000

Summary The IAU has not established any new time scales since 1991, but more recent IAU resolutions have redefined or clarified those already in use, with no loss of continuity. There are two classes of time scales used in astronomy, one based on the SI (atomic) second, the other based on the rotation of the Earth. The SI second has a simple definition that allows it to be used (in practice or in theory) in any reference system. Time scales based on the SI second include TAI and TT for practical applications, and TCG and TCB for theoretical developments. The latter are to be used for relativistically correct dynamical theories in the geocentric and barycentric reference systems, respectively. Closely related to these are two time scales, TDB and $\mathrm{T_{eph}}$, used in the current generation of ephemerides. Time scales based on the rotation of the Earth include mean and apparent sidereal time and UT1. Because of irregularities in the Earth's rotation, and its tidal deceleration, Earth-rotation-based time scales do not advance at a uniform rate, and they increasingly lag behind the SI-second-based time scales. UT1 is now defined to be a linear function of a quantity called the *Earth Rotation Angle*, θ. In the formula for mean sidereal time, θ now constitutes the "fast term" The widely disseminated time scale UTC is a hybrid: it advances by SI seconds but is subject to one-second corrections (*leap seconds*) to keep it within $0\overset{\mathrm{s}}{.}9$ of UT1. That procedure is now the subject of debate and there is a movement to eliminate leap seconds from UTC.

2.1 Different Flavors of Time

The phrase *time scale* is used quite freely in astronomical contexts, but there is sufficient confusion surrounding astronomical times scales that it is worthwhile revisiting the basic concept. A time scale is simply a well defined way of measuring time based on a specific periodic natural phenomenon. The definition of a time scale must provide a description of the phenomenon to be used (what defines a period, and under what conditions), the rate of advance (how many time units correspond to the natural period), and an initial epoch (the time reading at some identifiable event). For example, we could define a time scale where the swing of a certain kind of pendulum, in vacuum at sea level, defines one second, and where the time 00:00:00 corresponds to the transit of a specified star across a certain geographic meridian on an agreed-upon date.

As used in astronomy, a time scale is an idealization, a set of specifications written on a piece of paper. The instruments we call clocks, no matter how sophisticated or accurate, provide some imperfect approximation to the time scale they are meant to represent. In this sense, time scales are similar to spatial reference systems (see Chapter 3), which have precise definitions but various imperfect realizations. The parallels are not coincidental, since for modern high-precision applications we actually use space-time reference systems (see Chapter 1). All time scales are therefore associated with specific reference systems.

Two fundamentally different groups of time scales are used in astronomy. The first group of time scales is based on the second that is defined as part of the the Système International (SI) — the "atomic" second — and the second group is based on the rotation of the Earth. The SI second is defined as 9, 192, 631, 770 cycles of the radiation corresponding to the ground state hyperfine transition of Cesium 133 (BIPM 1998), and provides a very precise and constant rate of time measurement, at least for observers local to the apparatus in which such seconds are counted. The rotation of the Earth (length of day) is quite a different basis for time, since it is variable and has unpredictable components. It must be continuously monitored through astronomical observations, now done primarily with very long baseline [radio] interferometry (VLBI). The SI-based time scales are relatively new in the history of timekeeping, since they rely on atomic clocks first put into regular use in the 1950s. Before that, all time scales were tied to the rotation of the Earth. (Crystal oscillator clocks in the 1930s were the first artificial timekeeping mechanisms to exceed the accuracy of the Earth itself.) As we shall see, the ubiquitous use of SI-based time for modern applications has led to a conundrum about what the relationship between the two kinds of time should be in the future. Both kinds of time scales can be further subdivided into those that are represented by actual clock systems and those that are simply theoretical constructs.

General reviews of astronomical time scales are given in Seidelmann & Fukushima (1992) and Chapter 2 of the Explanatory Supplement (1992).

2.2 Time Scales Based on the SI Second

Let us first consider the times scales based on the SI second. As a simple count of cycles of microwave radiation from a specific atomic transition, the SI second can be implemented, at least in principle, by an observer anywhere. Thus, SI-based time scales can be constructed or hypothesized on the surface of the Earth, on other celestial bodies, on spacecraft, or at theoretically interesting locations in space, such as the solar system barycenter. According to relativity theory, clocks advancing by SI seconds may not appear to advance by SI seconds by an observer on another space-time trajectory. In general, there will be an observed difference in rate and possibly higher-order or periodic differences, depending on the relative trajectory of the clock and the observer and the gravitational fields involved. The precise conversion formulas can be mathematically complex, involving the positions and velocities not just of the clock and observer but also those of an ensemble of massive bodies (Earth, Sun, Moon, planets). These considerations also apply to coordinate time scales established for specific reference systems. The time-scale conversions are taken from the general 4-dimensional space-time transformation between the reference systems given by relativity theory (see Chapter 1). While the rate differences among these time scales may seem inconvenient, the universal use of SI units, including "local" SI seconds, means that the values of fundamental physical constants determined in one reference system can be used in another reference system *without* scaling factors.

Two SI-second-based times have already been mentioned in Chapter 1: these are *coordinate time* scales (in the terminology of General Relativity) for theoretical developments based on the Barycentric Celestial Reference System (BCRS) or the Geocentric Celestial Reference System (GCRS). These time scales are called, respectively, Barycentric Coordinate Time (TCB) and Geocentric Coordinate Time (TCG). With respect to a time scale based on SI seconds on the surface of the Earth, TCG advances at a rate 6.97×10^{-10} faster, while TCB advances at a rate 1.55×10^{-8} faster. TCB and TCG are not likely to come into common use for practical applications, but they are beginning to appear as the independent argument for some theoretical developments in dynamical astronomy (e.g., Moisson & Bretagnon (2001)). However, none of the current IAU recommended models used in the analysis of astrometric data use TCB or TCG as a basis, and neither time scale appears in the main pages of *The Astronomical Almanac.* This simply reflects the fact that there has not been enough time or motivation for a new generation of dynamical models to be fully developed within the IAU-recommended relativistic paradigm.

For practical applications, International Atomic Time (TAI) is a commonly used time scale based on the SI second on the Earth's surface at sea level (specifically, the rotating geoid). TAI is the most precisely determined time scale that is now available for astronomical use. This scale results from analyses by the Bureau International des Poids et Mesures (BIPM) in Sèvres, France, of data from atomic time standards of many countries, according to an agreed-upon algorithm. Although TAI was not officially introduced until 1972, atomic time scales have been available since 1956, and TAI may be extrapolated backwards to the period 1956–1971 (See Nelson et al. (2001) for a history of TAI). An interesting discussion of whether TAI should be considered a *coordinate time* or a kind of modified *proper time*[1] in the context of general relativity has been given by Guinot (1986). In any event, TAI is readily available as an integral number of seconds offset from UTC, which is extensively disseminated; UTC is discussed at the end of this chapter. The TAI offset from UTC is designated ΔAT = TAI–UTC. (For example, from 1999 through 2005, ΔAT = 32 s.) ΔAT increases by 1 s whenever a positive *leap second* is introduced into UTC (see below). The history of ΔAT values can be found on page K9 of *The Astronomical Almanac* and the current value can be found at the beginning of each issue of IERS Bulletin A (URL 5).

The astronomical time scale called Terrestrial Time (TT), used widely for geocentric and topocentric ephemerides (such as in *The Astronomical Almanac*), is defined to run at a rate of $(1 - \mathrm{L}_G)$ times that of TCG, where $\mathrm{L}_G = 6.969290134\times10^{-10}$. The rate factor applied to TCG to create TT means that TT runs at the same rate as a time scale based on SI seconds on the surface of the Earth. L_G is now considered a defining constant, not subject to further revision. Since TCG is a theoretical time scale that is not kept by any real clock, for practical purposes, TT can be considered an idealized form of TAI with an epoch offset: TT = TAI + $32\overset{s}{.}184$. This expresssion for TT preserves continuity with previously-used (now obsolete) "dynamical" time scales, Terrestrial Dynamical Time (TDT) and Ephemeris Time (ET). That is, ET → TDT → TT can be considered a single continuous time scale.

Important Note: The "standard epoch" for modern astrometric reference data, designated J2000.0, is expressed as a TT instant: J2000.0 is 2000 January 1, 12^{h} TT (JD 2451545.0 TT) at the geocenter.

The fundamental solar system ephemerides from the Jet Propulsion Laboratory (JPL) that are the basis for many of the tabulations in *The Astronomical Almanac* and other national almanacs were computed in a barycentric reference system and are distributed with the independent argument being a coordinate time scale called $T_{\rm eph}$ (Chapter 4 describes the JPL ephemerides). $T_{\rm eph}$ differs in rate from that of TCB, the IAU recommended time scale for barycentric developments; the rate of $T_{\rm eph}$ matches that of TT, on average, over the time span of the ephemerides. One may treat $T_{\rm eph}$ as functionally equivalent to Barycentric Dynamical Time (TDB), defined by the IAU in 1976 and 1979. Both are meant to be "time scales for equations of motion referred to the barycenter of the solar system" yet (loosely speaking) match TT in average rate. The original IAU definition of TDB specified that "there be only periodic variations" with respect to what we now call TT (the largest variation is 0.0016 s with a period of one year). It is now clear that this condition cannot be rigorously fulfilled in practice; see Standish (1998a) for a discussion of the issue and the distinction between TDB and $T_{\rm eph}$. Nevertheless, space coordinates obtained from the JPL ephemerides are consistent with TDB, and it has been said that "$T_{\rm eph}$ is what TDB was meant to be." Therefore, barycentric and heliocentric data derived from the JPL ephemerides are often tabulated with TDB shown as the time argument (as in *The Astronomical Almanac*), and TDB is the specified time argument for many of the equations presented in this circular.[2] Because $T_{\rm eph}$ ($\approx$TDB) is not based on the SI second, as is TCB, the values of parameters determined from or consistent with the JPL ephemerides will, in general, require scaling to convert them to SI-based units. This includes the length of the astronomical unit. Dimensionless quantities such as mass ratios are unaffected.

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[1] These terms are described in Chapter 1, p. 2.

The problem of defining relativistic time scales in the solar system has been treated by Brumberg & Kopeikin (1990); the paper is quite general but pre-dates the current terminology.

2.3 Time Scales Based on the Rotation of the Earth

Time scales that are based on the rotation of the Earth are also used in astronomical applications, such as telescope pointing, that depend on the geographic location of the observer. Greenwich sidereal time is the hour angle of the equinox measured with respect to the Greenwich meridian. Local sidereal time is the local hour angle of the equinox, or the Greenwich sidereal time plus the longitude (east positive) of the observer, expressed in time units. Sidereal time appears in two forms, mean and apparent, depending on whether the mean or true equinox is the reference point. The position of the mean equinox is affected only by precession while the true equinox is affected by both precession and nutation. The difference between true and mean sidereal time is the *equation of the equinoxes*, which is a complex periodic function with a maximum amplitude of about 1 s. Of the two forms, apparent sidereal time is more relevant to actual observations, since it includes the effect of nutation. Greenwich (or local) apparent sidereal time can be observationally obtained from the right ascensions of celestial objects transiting the Greenwich (or local) meridian.

Universal Time (UT) is also widely used in astronomy, and now almost always refers to the specific time scale UT1. Historically, Universal Time (formerly, Greenwich Mean Time) has been obtained from Greenwich sidereal time using a standard expression. In 2000, the IAU redefined UT1 to be a linear function of the *Earth Rotation Angle*, θ, which is the geocentric angle between two directions in the equatorial plane called, respectively, the Celestial Intermediate Origin (CIO) and the Terrestrial Intermediate Origin (TIO) (res. B1.8 of 2000[3]). The TIO rotates with the Earth, while the CIO has no instantaneous rotation around the Earth's axis, so that θ is a direct measure of the Earth's rotational motion: $\dot{\theta} = \omega$, the Earth's average angular velocity of rotation. See Chapter 6 for a more complete description of these new reference points. The definition of UT1 based on sidereal time is still widely used, but the definition based on θ is becoming more common for precise applications. In fact, the two definitions are equivalent, since the expression for sidereal time as a function of UT1 is itself now based on θ.

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[2] The IAU Working Group on Nomenclature for Fundamental Astronomy is considering a recommendation to correct the definition of TDB so that a distinction between TDB and $T_{\rm eph}$ would no longer be necessary; TDB would be a linear function of TCB with a rate as close to that of TT as possible over the time span of the ephemeris to which it applies.

Since they are mathematically linked, sidereal time, θ, and UT1 are all affected by variations in the Earth's rate of rotation (length of day), which are unpredictable and must be routinely measured through astronomical observations. The lengths of the sidereal and UT1 seconds, and the value of $\dot{\theta}$, are therefore not precisely constant when expressed in a uniform time scale such as TT. The accumulated difference in time measured by a clock keeping SI seconds on the geoid from that measured by the rotation of the Earth is $\Delta T = \mathrm{TT{-}UT1}$. A table of observed and extrapolated values of ΔT is given in *The Astronomical Almanac* on page K9. The long-term trend is for ΔT to increase gradually because of the tidal deceleration of the Earth's rotation, which causes UT1 to lag increasingly behind TT.

In predicting the precise times of topocentric phenomena, like solar eclipse contacts, both TT and UT1 come into play. Therefore, assumptions have to be made about the value of ΔT at the time of the phenomenon. Alternatively, the circumstances of such phenomena can be expressed in terms of an imaginary system of geographic meridians that rotate uniformly about the Earth's axis (ΔT is assumed zero, so that UT1=TT) rather than with the real Earth; the real value of ΔT then does not need to be known when the predictions are made. The zero-longitude meridian of the uniformly rotating system is called the *ephemeris meridian*. As the time of the phenomenon approaches and the value of ΔT can be estimated with some confidence, the predictions can be related to the real Earth: the uniformly rotating system is 1.002738 ΔT east of the real system of geographic meridians. (The 1.002738 factor converts a UT1 interval to the equivalent Earth Rotation Angle — i.e., the sidereal/solar time ratio.)

2.4 Coordinated Universal Time (UTC)

The worldwide system of civil time is based on Coordinated Universal Time (UTC), which is now ubiquitous and tightly synchronized. (This is the de facto situation; most nations' legal codes, including that of the U.S., do not mention UTC specifically.) UTC is a hybrid time scale, using the SI second on the geoid as its fundamental unit, but subject to occasional 1-second adjustments to keep it within $0^{\mathrm{s}}.9$ of UT1. Such adjustments, called *leap seconds*, are normally introduced at the end of June or December, when necessary, by international agreement. Tables of the remaining difference, UT1–UTC, for various dates are published by the International Earth Rotation and Reference System Service (IERS), at URL 5. Both past observations and predictions are available. DUT1, an approximation to UT1–UTC, is transmitted in code with some radio time signals, such as those from WWV. As previously discussed in the context of TAI, the difference ΔAT = TAI–UTC is an integral number of seconds, a number that increases by 1 whenever a (positive) leap second is introduced into UTC. That is, UTC and TAI share the same seconds ticks, they are just labeled differently.

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[3] In the resolution, these points are called the Celestial Ephemeris Origin (CEO) and the Terrestrial Ephemeris Origin (TEO). The change in terminology has been recommended by the IAU Working Group on Nomenclature for Fundamental Astronomy and will probably be adopted at the 2006 IAU General Assembly.

Clearly UT1–UTC and ΔT must be related, since they are both measures of the natural "error" in the Earth's angle of rotation at some date. In fact, $\Delta T = 32.^{\mathrm{s}}184 + \Delta\mathrm{AT} - (\mathrm{UT1}-\mathrm{UTC})$.

For the user, then, UTC, which is widely available from GPS, radio broadcast services, and the Internet, is the practical starting point for computing any of the other time scales described above. For the SI-based time scales, we simply add the current value of ΔAT to UTC to obtain TAI. TT is then just $32.^{\mathrm{s}}184$ seconds ahead of TAI. The theoretical time scales TCG, TCB, TDB, and $\mathrm{T}_{\mathrm{eph}}$ can be obtained from TT using the appropriate mathematical formulas. For the time scales based on the rotation of the Earth, we again start with UTC and add the current value of UT1–UTC to obtain UT1. The various kinds of sidereal time can then be computed from UT1 using standard formulas.

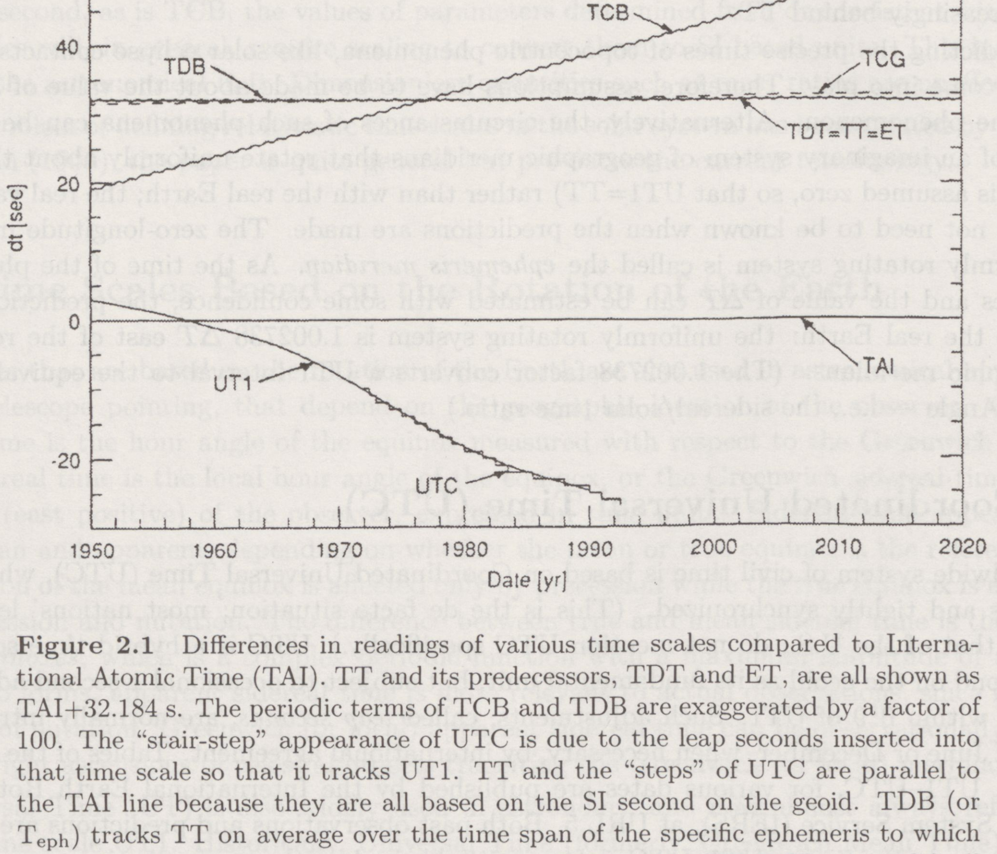

Figure 2.1 Differences in readings of various time scales compared to International Atomic Time (TAI). TT and its predecessors, TDT and ET, are all shown as TAI+32.184 s. The periodic terms of TCB and TDB are exaggerated by a factor of 100. The "stair-step" appearance of UTC is due to the leap seconds inserted into that time scale so that it tracks UT1. TT and the "steps" of UTC are parallel to the TAI line because they are all based on the SI second on the geoid. TDB (or $\mathrm{T}_{\mathrm{eph}}$) tracks TT on average over the time span of the specific ephemeris to which it applies. Note the instant at the beginning of 1977 when TT, TCB, and TCG all had the same value. The figure is from Seidelmann & Fukushima (1992).

2.5 To Leap or Not to Leap

Because of the widespread and increasing use of UTC for applications not considered three decades ago — such as precisely time-tagging electronic fund transfers and other networked business transactions — the addition of leap seconds to UTC at unpredictable intervals creates technical problems and legal issues for service providers. There is now a movement to relax the requirement that UTC remain within 0.9 seconds of UT1. The issue is compounded by the unavoidable scientific fact that the Earth's rotation is slowing due to tidal friction, so that the rate of addition of leap seconds to UTC must inevitably increase. Aside from monthly, annual, and decadal variations, the Earth's angular velocity of rotation is decreasing linearly, which means that the accumulated lag in UT1 increases quadratically; viewed over many centuries, the ΔT curve is roughly a parabola. The formulas for sidereal time, and length of the old *ephemeris second* to which the SI second was originally calibrated, are based on the average (assumed fixed) rate of Earth rotation of the mid-1800s (Nelson et al. 2001). All of our modern timekeeping systems are ultimately based on what the Earth was doing a century and a half ago.

An IAU Working Goup on the Redefinition of Universal Time Coordinated (UTC) was established to consider the leap second problem and recommend a solution, working with the IERS, the International Union of Radio Science (URSI), the Radiocommunication Sector of the International Telecommunications Union (ITU-R), the International Bureau for Weights and Measures (BIPM), and the relevant navigational agencies (res. B2 of 2000). Possibilities include: using TAI for technical applications instead of UTC; allowing UT1 and UTC to diverge by a larger amount (e.g., 10 or 100 seconds) before a multi-second correction to UTC is made; making a variable correction to UTC at regularly scheduled dates; eliminating the corrections to UTC entirely and allowing UTC and UT1 to drift apart; or changing the definition of the SI second. No solution is ideal (including the status quo) and each of these possibilities has its own problems. For example, if we keep leap seconds, or a less frequent multi-second correction, can current systems properly time-tag the date and time of an event that occurs *during* the correction? Does a time scale that diverges from UT1 provide a legally acceptable representation of civil time? If corrections are made less frequently, will the possibility of technical blunders increase? If leap seconds are eliminated, won't natural phenomena such as sunrise and sunset eventually fall out of sync with civil time? How do we find all the existing computer code that assumes $|\mathrm{UT1}-\mathrm{UTC}| \leq 0.9\,\mathrm{s}$? The matter is now being considered by the ITU-R, where a working group has proposed eliminating leap seconds from UTC entirely. Contact Dr. Dennis McCarthy, U.S. Naval Observatory, dmc@maia.usno.navy.mil, for a copy of a report or if you wish to comment. In any event, it would take a number of years for any proposed change to take place because of the many institutions and international bodies that would have to be involved.

For scientific instrumentation, the use of TAI — which is free of leap seconds — has much to recommend it. Its seconds can be easily synchronized to those of UTC (only the labels of the seconds are different). It is straightforward to convert from TAI to any of the other time scales. Use of TAI provides an internationally recognized time standard and avoids the need to establish an instrument-specific time scale when continuity of time tags is a requirement.

2.6 Formulas

2.6.1 Formulas for Time Scales Based on the SI Second

For the SI-based time scales, the event tagged 1977 January 1, 00:00:00 TAI (JD 2443144.5 TAI) at the geocenter is special. At that event, the time scales TT, TCG, and TCB all read 1977 January 1, 00:00:32.184 (JD 2443144.5003725). (The $32^{\mathrm{s}}.184$ offset is the estimated difference between TAI and the old Ephemeris Time scale.) This event will be designated t_0 in the following; it can be represented in any of the time scales, and the context will dictate which time scale is appropriate.

From the perspective of a user, the starting point for computing all the time scales is Coordinated Universal Time (UTC). From UTC, we can immediately get International Atomic Time (TAI):

$$\mathrm{TAI} = \mathrm{UTC} + \Delta\mathrm{AT} \tag{2.1}$$

where ΔAT, an integral number of seconds, is the accumulated number of leap seconds applied to UTC.

The astronomical time scale Terrestrial Time (TT) is defined by the epoch t_0 and its IAU-specified rate with respect to Geocentric Coordinate Time (TCG):

$$\frac{d\mathrm{TT}}{d\mathrm{TCG}} = 1 - \mathrm{L}_G \quad \text{where} \quad \mathrm{L}_G = 6.969290134{\times}10^{-10} \text{ (exactly)} \tag{2.2}$$

from which we obtain

$$\mathrm{TT} = \mathrm{TCG} - \mathrm{L}_G\,(\mathrm{TCG} - t_0) \tag{2.3}$$

However, TCG is a theoretical time scale, not kept by any real clock system, so in practice,

$$\mathrm{TT} = \mathrm{TAI} + 32\overset{\mathrm{s}}{.}184 \tag{2.4}$$

and we obtain TCG from TT.

The relationship between TCG and Barycentric Coodinate Time (TCB) is more complex. TCG and TCB are both coordinate time scales, to be used with the geocentric and barycentric reference systems (the GCRS and BCRS), respectively. The exact formula for the relationship between TCG and TCB is given in res. B1.5 of 2000, recommendation 2. For a given TCB epoch, we have

$$\mathrm{TCG} = \mathrm{TCB} - \frac{1}{c^2}\int_{t_0}^{\mathrm{TCB}} \left(\frac{v_e^2}{2} + U_{ext}(\mathbf{x}_e)\right) dt - \frac{\mathbf{v}_e}{c^2}\cdot(\mathbf{x} - \mathbf{x}_e) + \cdots \tag{2.5}$$

where c is the speed of light, $\mathbf{x}_e$ and $\mathbf{v}_e$ are the position and velocity vectors of the Earth's center with respect to the solar system barycenter, and U_{ext} is the Newtonian potential of all solar system bodies apart from the Earth. The integral is carried out in TCB since the positions and motions of the Earth and other solar system bodies are represented (ideally) as functions of TCB. The last term on the right contains the barycentric position vector of the point of interest, $\mathbf{x}$, and will be zero for the geocenter, as would normally be the case. The omitted terms are of maximum order c^{-4}. Note that the transformation is ephemeris-dependent, since it is a function of the time series of $\mathbf{x}_e$ and $\mathbf{v}_e$ values. The result is a "time ephemeris" associated with every spatial ephemeris of solar system bodies expressed in TCB. It is to be expected that ephemeris developers will supply appropriate time conversion algorithms (software) to allow the positions and motions of solar system bodies to be retrieved for epochs in conventional time scales such as TT or TAI. It is unlikely that ordinary ephemeris users will have to compute eq. (2.5) on their own.

The functional form of the above expressions may seem backwards for practical applications; that is, they provide TCG from TCB and TT from TCG. These forms make sense, however, when one considers how an ephemeris of a solar system body (or bodies) or a spacecraft is developed. The equations of motion for the body (or bodies) of interest are expressed in either the barycentric or geocentric system as a function of some independent coordinate time argument. For barycentric equations of motion, expressed in SI units, we would be tempted immediately to identify this time argument with TCB. Actually, however, the association of the time argument with TCB is not automatic; it comes about only when the solution of the equations of motion is made to satisfy the boundary conditions set by the ensemble of real observations of various kinds. Generally, these observations will be time-tagged in UTC, TAI, or TT (all of which are based on the SI second on the geoid) and these time tags must be associated with the time argument of the ephemeris. The above formulas can be used to make that association, which then allows the ephemeris to be fit to the observations. (More precisely, the space-time coordinates of the observation events must be transformed to the BCRS.) As a consequence, the time argument of the ephemeris becomes a realization of TCB. The fit of the computed ephemeris to observations usually proceeds iteratively, and every iteration of the spatial ephemeris produces a new time ephemeris. With each iteration, the spatial coordinates of the ephemeris become better grounded in reality (as represented by the observations) and the time coordinate becomes a better approximation to TCB. Viewed from this computational perspective, the ephemeris and its time argument are the starting point of the process and the sequence TCB → TCG → TT makes sense.

One can compute an ephemeris and fit it to observations using other formulas for the time scale conversions. A completely valid and precise ephemeris can be constructed in this way, but its independent time argument could not be called TCB. The values of various constants used in, or derived from, such an ephemeris would also not be SI-based and a conversion factor would have to be applied to convert them to or from SI units. Such is the case with the solar system Development Ephemeris (DE) series from the Jet Propulsion Laboratory. DE405 is now the consensus standard for solar system ephemerides and is described in Chapter 4. The DE series dates back to the 1960s, long before TCB and TCG were defined, and its independent time argument is now called $\mathrm{T_{eph}}$. $\mathrm{T_{eph}}$ can be considered to be TCB with a rate factor applied. Or, as mentioned above, $\mathrm{T_{eph}}$ can be considered to be functionally equivalent to the time scale called TDB. Both $\mathrm{T_{eph}}$ and TDB advance, on average, at the same rate as TT. This arrangement makes accessing the DE ephemerides straightforward, since for most purposes, TT can be used as the input argument with little error. The total error in time in using TT as the input argument is <2 ms, which for the geocentric position of the Moon would correspond to an angular error of <1 mas. When more precision is required, the following formula can be used:

$$\begin{aligned} \mathrm{T_{eph}} \approx \mathrm{TDB} \approx \mathrm{TT} \; &+ \; 0.001657 \sin(628.3076\,T + 6.2401) \\ &+ \; 0.000022 \sin(575.3385\,T + 4.2970) \\ &+ \; 0.000014 \sin(1256.6152\,T + 6.1969) \\ &+ \; 0.000005 \sin(606.9777\,T + 4.0212) \\ &+ \; 0.000005 \sin(52.9691\,T + 0.4444) \\ &+ \; 0.000002 \sin(21.3299\,T + 5.5431) \\ &+ \; 0.000010\,T \sin(628.3076\,T + 4.2490) \; + \cdots \end{aligned} \tag{2.6}$$

where the coefficients are in seconds, the angular arguments are in radians, and T is the number of Julian centuries of TT from J2000.0: $T = (\mathrm{JD(TT)} - 2451545.0)/36525$. The above is a truncated form of a much longer and more precise series given by Fairhead & Bretagnon (1990). The maximum error in using the above formula is about 10 μs from 1600 to 2200; that is, its precision is more than two orders of magnitude better than the approximation $\mathrm{T_{eph}} \approx \mathrm{TDB} \approx \mathrm{TT}$. For even more precise applications, the series expansion by Harada & Fukushima (2003) is recommended.

A word of caution: The idea that "$\mathrm{T_{eph}}$ and TDB advance, on average, at the same rate as TT" is problematic. The independent time argument of a barycentric ephemeris (whether considered to be $\mathrm{T_{eph}}$, TDB, or TCB) has a large number of periodic components with respect to TT. Some of the periods are quite long, and may extend beyond the time period of the ephemeris. Thus, the "average rate" of the time argument of the ephemeris, with respect to TT, depends on the averaging method and the time span considered. Differences in rate of some tens of microseconds per century are possible (Fairhead & Bretagnon 1990). These rate ambiguities are probably unimportant (amounting to fractional errors of only $\sim 10^{-14}$) for retrieving positions and velocities from the ephemeris but do affect pulsar timing that is reduced to the barycentric time scale.

2.6.2 Formulas for Time Scales Based on the Rotation of the Earth

For the time scales that are based on the rotation of the Earth, we again start with UTC.

$$\begin{aligned} \mathrm{UT1} &= \mathrm{UTC} + (\mathrm{UT1}-\mathrm{UTC}) \qquad (2.7) \\ &\approx \mathrm{UTC} + \mathrm{DUT1} \qquad (2.8) \end{aligned}$$

where DUT1 is a broadcast approximation to UT1–UTC (precision $\pm 0\overset{\mathrm{s}}{.}1$). We also have

$$\mathrm{UT1} = \mathrm{TT} - \Delta T \tag{2.9}$$

where $\Delta T = 32\overset{\mathrm{s}}{.}184 + \Delta\mathrm{AT} - (\mathrm{UT1}-\mathrm{UTC})$. The most recent values of UT1–UTC and ΔAT are listed in IERS Bulletin A (URL 5). Values of ΔT are listed in *The Astronomical Almanac* on page K9.

The Earth Rotation Angle, θ, is

$$\theta = 0.7790572732640 + 1.00273781191135448\, D_U \tag{2.10}$$

where D_U is the number of UT1 days from 2000 January 1, 12^{h} UT1: $D_U = \mathrm{JD(UT1)} - 2451545.0$. The angle θ is given in terms of rotations (units of 2π radians or 360°). The above rate coefficient gives an Earth rotation period of 86164.0989036903511 seconds of UT1. If we consider this to be the adopted average rotation period of the Earth in SI seconds, it is consistent with the nominal mean angular velocity of Earth rotation, $\omega = 7.292115 \times 10^{-5}$ radian s^{-1}, used by the International Association of Geodesy. The above expression is taken directly from note 3 to res. B1.8 of 2000. An equivalent form of this expression (if the integral number of rotations is neglected) that is usually more numerically precise is

$$\theta = 0.7790572732640 + 0.00273781191135448\, D_U + \mathrm{frac(JD(UT1))} \tag{2.11}$$

where frac(JD(UT1)) is the fractional part of the UT1 Julian date, i.e., JD(UT1) modulus 1.0. Then Greenwich mean sidereal time in seconds is

$$\begin{aligned} \mathrm{GMST} = 86400 \cdot \theta + {} & (0.014506 + 4612.156534\,T + 1.3915817\,T^2 \\ & -0.00000044\,T^3 - 0.000029956\,T^4 - 0.0000000368\,T^5)/15 \qquad (2.12) \end{aligned}$$

where T is the number of centuries of TDB (equivalently for this purpose, TT) from J2000.0: $T = (\mathrm{JD(TDB)} - 2451545.0)/36525$. The polynomial in parentheses is the accumulated precession of the equinox in right ascension, in arcseconds, as given for the P03 solution (eq. 42) in Capitaine et al. (2003). Note that two time scales are now required to compute sidereal time: in the "fast term", θ is a function of UT1, while in the remaining terms, T is expressed in TDB (or TT).

To obtain Greenwich apparent sidereal time in seconds, we must add the *equation of the equinoxes*:

$$\mathrm{GAST} = \mathrm{GMST} + \mathcal{E}_{\Upsilon}/15 \tag{2.13}$$

which accounts for the motion of the equinox due to nutation. An extended series is now used for the equation of the equinoxes. The new series includes so-called *complementary terms* and more fully accounts for the accumulated effect of combined precession and nutation on the position of the equinox. The equation of the equinoxes in arcseconds is

$$\begin{aligned}\mathcal{E}_{\Upsilon} = \; & \Delta\psi \,\cos\epsilon \\ + \; & 0.00264096 \,\sin(\Omega) \\ + \; & 0.00006352 \,\sin(2\Omega) \\ + \; & 0.00001175 \,\sin(2F - 2D + 3\Omega) \\ + \; & 0.00001121 \,\sin(2F - 2D + \Omega) \\ - \; & 0.00000455 \,\sin(2F - 2D + 2\Omega) \\ + \; & 0.00000202 \,\sin(2F + 3\Omega) \\ + \; & 0.00000198 \,\sin(2F + \Omega) \\ - \; & 0.00000172 \,\sin(3\Omega) \\ - \; & 0.00000087\, T \,\sin(\Omega) + \cdots\end{aligned} \tag{2.14}$$

where $\Delta\psi$ is the nutation in longitude, in arcseconds; ϵ is the mean obliquity of the ecliptic; and F, D, and Ω are fundamental luni-solar arguments. All of these quantities are functions of TDB (or TT); see Chapter 5 for expressions (esp. eqs. 5.12, 5.15, & 5.19). The above series is a truncated form of a longer series given in the IERS Conventions (2003), but should be adequate for almost all practical applications.

Local mean sidereal time (LMST) and local apparent sidereal time (LAST) in seconds can then be computed respectively from

$$\text{LMST} = \text{GMST} + \left(\frac{3600}{15}\right)\lambda \quad \text{and} \quad \text{LAST} = \text{GAST} + \left(\frac{3600}{15}\right)\lambda \tag{2.15}$$

where λ is the longitude of the place of interest, in degrees, positive for places east of Greenwich.

In the above, "Greenwich" actually refers to a plane containing the geocenter, the Celestial Intermediate Pole (CIP), and the point called the Terrestrial Intermediate Origin (TIO). These concepts are described in Chapters 5 and 6. Loosely, the CIP is the rotational pole, defined by the precession and nutation theories. For astronomical purposes, the TIO can be considered to be a point on the rotational equator (the plane orthogonal to the CIP) essentially fixed at geodetic longitude 0. Strictly, then, the longitude λ should be measured around the axis of the CIP from the TIO to the location of interest. Because of polar motion, the pole of the conventional system of geodetic coordinates is not at the CIP so the longitude we need is not quite the same as the geodetic longitude. The longitude, in degrees, to be used in eq. 2.15 is

$$\lambda = \lambda_G + (x_p \sin\lambda_G + y_p \cos\lambda_G)\,\tan\phi_G \,/3600 \tag{2.16}$$

where λ_G and ϕ_G are the usual geodetic longitude and latitude of the place, with λ_G in degrees (north latitudes and east longitudes are positive); and x_p and y_p are the coordinates of the pole (CIP) with respect to the geodetic system, in arcseconds (x_p and y_p can be a few tenths of an arcsecond). The geodetic system is formally the International Terrestrial Reference System (ITRS), which matches WGS-84 (available from GPS) to several centimeters. The local meridian assumed by the formula for LAST, using the longitude λ, passes through the local zenith (orthogonal to the local surface of the WGS-84 ellipsoid) and the north and south celestial poles — close to but not through the local geodetic north and south points. This is the meridian that all stars with apparent topocentric right ascension equal to LAST will pass over at time UT1. More information can be found in sections 6.4, 6.5.2, and 6.5.4.

The above formulas are entirely geometric. Not described are astronomical latitude and longitude, which are based on the local direction of gravity. Astronomical latitude and longitude are affected by the deflection of the vertical caused by permanent gravitational anomalies and, at a much lower level, semidiurnal tides. Astronomical latitude and longitude must be corrected for such effects to obtain geodetic latitude and longitude.

Chapter 3

The Fundamental Celestial Reference System

Relevant IAU resolutions: A4.VI, A4.VII of 1991; B5 of 1994; B2 of 1997; B1.2 of 2000

Summary Reference data for positional astronomy, such as the data in astrometric star catalogs or barycentric planetary ephemerides, are now specified within the International Celestial Reference System (ICRS). The ICRS is a coordinate system whose origin is at the solar system barycenter and whose axis directions are effectively defined by the adopted coordinates of 212 extragalactic radio sources observed by VLBI. These radio sources (quasars and active galactic nuclei) are assumed to have no observable intrinsic angular motions. Thus, the ICRS is a "space-fixed" system (more precisely, a kinematically non-rotating system) without an associated epoch. However, the ICRS closely matches the conventional dynamical system defined by the Earth's mean equator and equinox of J2000.0; the alignment difference is at the 0.02 arcsecond level, negligible for many applications.

Strictly speaking, the ICRS is somewhat of an abstraction, a coordinate system that perfectly satisfies a list of criteria. The list of radio source positions that define it for practical purposes is called the International Celestial Reference Frame (ICRF). In the terminology that is now commonly used, a *reference system* like the ICRS is "realized" by a *reference frame* like the ICRF, and there can be more than one such realization. In the case of the ICRS, there is, in fact, a second, lower-accuracy realization for work at optical wavelengths, called the Hipparcos Celestial Reference Frame (HCRF). The HCRF is composed of the positions and proper motions of the astrometrically well-behaved stars in the Hipparcos catalog.

Astrometric data referred to the ICRS is becoming more common, with new catalogs now available in the optical, infrared, and radio.

The ICRS is itself a specific example of a Barycentric Celestial Reference System, incorporating the relativistic metric specified in res. B1.3 of 2000 for solar system barycentric coordinate systems. In other words, the ICRS provides the orientation of the BCRS axes.

3.1 The ICRS, the ICRF, and the HCRF

The fundamental celestial reference system for astronomical applications is now the International Celestial Reference System (ICRS), as provided in res. B2 of 1997. The ICRS is a coordinate system with its origin at the solar system barycenter and axis directions that are fixed with respect to distant objects in the universe; it is to be used to express the positions and motions of stars, planets, and other celestial objects. Its relativistic basis is defined by res. B1.3 of 2000; in the words of that resolution, it is a Barycentric Celestial Reference System (BCRS), and as such its axes are *kinematically non-rotating* (see Chapter 1). To establish the ICRS as a practical system, the IAU specified a set of distant benchmark objects, observable at radio wavelengths, whose adopted coordinates effectively define the directions of the ICRS axes. This "realization" of the ICRS, called the International Celestial Reference Frame (ICRF), is a set of high accuracy positions of extragalactic radio sources measured by very long baseline interferometry (Ma & Feissel 1997; Ma et al. 1998). The ICRS is realized at optical wavelengths — but at lower accuracy — by the Hipparcos Celestial Reference Frame (HCRF), consisting of the *Hipparcos Catalogue* (ESA 1997) of star positions and motions, with certain exclusions (res. B1.2 of 2000). The coordinates of the ICRF radio sources and HCRF stars are given relative to the ICRS origin at the solar system barycenter, and a number of transformations are required to obtain the coordinates that would be observed from a given location on Earth at a specific date and time.

Although the directions of the ICRS coordinate axes are not defined by the kinematics of the Earth, the ICRS axes (as implemented by the ICRF and HCRF) closely approximate the axes that would be defined by the mean Earth equator and equinox of J2000.0 (to within about 0.02 arcsecond), if the latter is considered to be a barycentric system. Because the ICRS axes are meant to be "space fixed", i.e., kinematically non-rotating, there is no date associated with the ICRS. Furthermore, since the defining radio sources are assumed to be so distant that their angular motions, seen from Earth, are negligible, there is no epoch associated with the ICRF. It is technically incorrect, then, to say that the ICRS is a "J2000.0 system", even though for many current data sources, the directions in space defined by the equator and equinox of J2000.0 and the ICRS axes are the same to within the errors of the data.

The ICRS, with its origin at the solar system barycenter and "space fixed" axis directions, is meant to represent the most appropriate coordinate system currently available for expressing reference data on the positions and motions of celestial objects.

The IAU Working Group on Nomenclature for Fundamental Astronomy has recommended the following definitions for the ICRS and ICRF:

> **International Celestial Reference System (ICRS):** The idealized barycentric coordinate system to which celestial positions are referred. It is kinematically non-rotating with respect to the ensemble of distant extragalactic objects. It has no intrinsic orientation but was aligned close to the mean equator and dynamical equinox of J2000.0 for continuity with previous fundamental reference systems. Its orientation is independent of epoch, ecliptic or equator and is realized by a list of adopted coordinates of extragalactic sources.
>
> **International Celestial Reference Frame (ICRF):** A set of extragalactic objects whose adopted positions and uncertainties realize the ICRS axes and give the uncertainties of the axes. It is also the name of the radio catalogue whose 212 defining sources are currently the most accurate realization of the ICRS. Note that the orientation of the ICRF catalogue was carried over from earlier IERS radio catalogs and was within the

errors of the standard stellar and dynamic frames at the time of adoption. Successive revisions of the ICRF are intended to minimize rotation from its original orientation. Other realizations of the ICRS have specific names (e.g. Hipparcos Celestial Reference Frame).

3.2 Background: Reference Systems and Reference Frames

The terminology that has become standard over the past decade or so distinguishes between a *reference system* and a *reference frame*. A *reference system* is the complete specification of how a celestial coordinate system is to be formed. Both the origin and the orientation of the fundamental planes (or axes) are defined. A reference system also incorporates a specification of the fundamental models needed to construct the system; that is, the basis for the algorithms used to transform between observable quantities and reference data in the system. A *reference frame*, on the other hand, consists of a set of identifiable fiducial points on the sky along with their coordinates, which serves as the practical realization of a reference system.

For example, the fundamental plane of an astronomical reference system has conventionally been the extension of the Earth's equatorial plane, at some date, to infinity. Declination is the angular distance north or south of this plane, and right ascension is the angular distance measured eastward along the equator from some defined reference point. This reference point, the right ascension origin, has traditionally been the *equinox*: the point at which the Sun, in its yearly circuit of the celestial sphere, crosses the equatorial plane moving from south to north. The Sun's apparent yearly motion lies in the *ecliptic*, the plane of the Earth's orbit. The equinox, therefore, is a direction in space along the nodal line defined by the intersection of the ecliptic and equatorial planes; equivalently, on the celestial sphere, the equinox is at one of the two intersections of the great circles representing these planes. Because both of these planes are moving, the coordinate systems that they define must have a date associated with them; such a reference system must therefore be specified as "the equator and equinox of (some date)".

Of course, such a reference system is an idealization, because the theories of motion of the Earth that define how the two planes move are imperfect. In fact, the very definitions of these planes are problematic for high precision work. Even if the fundamental planes of a reference system are defined without any reference to the motions of the Earth, there is no way magically to paint them on the celestial sphere at any particular time. Therefore, in practice, we use a specific reference frame — a set of fiducial objects with assigned coordinates — as the practical representation of an astronomical reference system. The scheme is completely analogous to how terrestrial reference systems are established using survey control stations (geodetic reference points) on the Earth's surface.

Most commonly, a reference frame consists of a catalog of precise positions (and motions, if measurable) of stars or extragalactic objects as seen from the solar system barycenter at a specific epoch (now usually "J2000.0", which is 12^{h} TT on 1 January 2000). Each object's instantaneous position, expressed as right ascension and declination, indicates the object's angular distance from the catalog's equator and origin of right ascension. Any two such objects in the catalog (if they are not coincident or antipodal) therefore uniquely orient a spherical coordinate system on the sky — a reference frame.

A modern astrometric catalog contains data on a large number of objects (N), so the coordinate system is vastly overdetermined. The quality of the reference frame defined by a catalog depends on the extent to which the coordinates of all possible pairs of objects ($\approx N^2/2$) serve to define the identical equator and right ascension origin, within the expected random errors. Typically, every catalog contains *systematic errors*, that is, errors in position that are similar for objects that are in the same area of the sky, or are of the same magnitude (flux) or color (spectral index). Systematic errors mean that the reference frame is warped, or is effectively different for different classes of objects. Obviously, minimizing systematic errors when a catalog is constructed is at least as important as minimizing the random errors.

To be useful, a reference frame must be implemented at the time of actual observations, and this requires the computation of the apparent coordinates of the catalog objects at arbitrary dates and times. The accuracy with which we know the motions of the objects across the sky is an essential factor in this computation. Astrometric star catalogs list *proper motions*, which are the projection of each star's space motion onto the celestial sphere, expressed as an angular rate in right ascension and declination per unit time. Because the tabulated proper motions are never perfect, any celestial reference frame deteriorates with time. Moreover, systematic errors in the proper motions can produce time-dependent warpings and spurious rotations of the frame. Therefore, the accuracy and consistency of the proper motions are critical to the overall quality, utility, and longevity of reference frames defined by stars. Even reference frames defined by extragalactic objects, which are usually considered to have zero proper motion, may deteriorate, because many of these objects show small apparent motions that are artifacts of their emission mechanisms.

The positions of solar system objects can also be used to define a reference frame. For each solar system body involved, an *ephemeris* (pl. *ephemerides*) is used, which is simply a table of the celestial coordinates of the body as a function of time (or an algorithm that yields such a table). A reference frame defined by the ephemerides of one or more solar system bodies is called a *dynamical reference frame*. Because the ephemerides used incorporate the motion of the Earth as well as that of the other solar system bodies, dynamical reference frames embody in a very fundamental way the moving equator and ecliptic, hence the equinox. They have therefore been used to correct the orientation of star catalog reference frames (the star positions were systematically adjusted) on the basis of simultaneous observations of stars and planets. In a sense, the solar system is used as a gyrocompass. However, dynamical reference frames are not very practical for establishing a coordinate system for day-to-day astronomical observations.

Descriptions of reference frames and reference systems often refer to three coordinate axes, which are simply the set of right-handed cartesian axes that correspond to the usual celestial spherical coordinate system. The xy-plane is the equator, the z-axis points toward the north celestial pole, and the x-axis points toward the origin of right ascension. Although in principle this allows us to specify the position of any celestial object in rectangular coordinates, the distance scale (based on stellar parallaxes) is not established to high precision beyond the solar system. What a reference system actually defines is the way in which the two conventional astronomical *angular* coordinates, right ascension and declination, overlay real observable points in the sky. (See eqs. 5.1 & 5.2 for the conversion between rectangular and spherical celestial coordinates.)

3.3 Recent Developments

The establishment of celestial reference systems is coordinated by the IAU. The previous astronomical reference system was based on the equator and equinox of J2000.0 determined from observations of planetary motions, together with the IAU (1976) System of Astronomical Constants and related algorithms (Kaplan 1982). The reference frame that embodied this system for practical purposes was the Fifth Fundamental Catalogue (FK5). The FK5 is a catalog of 1535 bright stars (to magnitude 7.5), supplemented by a fainter extension of 3117 additional stars (to magnitude 9.5), compiled at the Astronomische Rechen-Institut in Heidelberg (Fricke et al. 1988; Fricke, et al. 1991). The FK5 was the successor to the FK3 and FK4 catalogs, all of which were based on meridian observations taken in the visual band — many such observations were, in fact, taken by eye. The formal uncertainties in the star positions of the FK5 at the time of its publication in 1988 were about 30–40 milliarcseconds over most of the sky, but the errors are considerably worse when systematic trends are taken into account.

In recent years, the most precise wide-angle astrometry has been conducted not in the optical regime but at radio wavelengths, involving the techniques of very long baseline interferometry (VLBI) and pulsar timing. Uncertainties of radio source positions listed in all-sky VLBI catalogs are now typically less than one milliarcsecond, and often a factor of ten better. Furthermore, because these radio sources are very distant extragalactic objects (mostly quasars) that are not expected to show measurable intrinsic motion, a reference frame defined by VLBI positions should be "more inertial" (less subject to spurious rotation) than a reference frame defined by galactic objects such as stars or pulsars. The VLBI catalogs do have the disadvantage that their origin of right ascension is somewhat arbitrary; there is no real equinox in VLBI catalogs, since VLBI has little sensitivity to the ecliptic plane. However, this problem has turned out to be more conceptual than practical, since methods have been developed to relate the VLBI right ascension origin to the equinox as conventionally defined.

Because of these considerations, since the mid 1980s, astronomical measurements of the Earth's rotation — from which astronomical time is determined — have depended heavily on VLBI, with classical methods based on star transits being phased out. Hence the situation evolved to where the definition of the fundamental astronomical reference frame (the FK5) became irrelevant to some of the most precise and important astrometric measurements. VLBI revealed, in addition, that the models of the Earth's precession and nutation that were part of the old system were inadequate for modern astrometric precision. In particular, the "constant of precession" — a measurement of the long-term rate of change of the orientation of the Earth's axis in space — had been overestimated by about 0.3 arcseconds per century. Moreover, the success of the European Space Agency's Hipparcos astrometric satellite, launched in 1989, promised to provide a new, very accurate set of star coordinates in the optical regime.

Thus, beginning in 1988, a number of IAU working groups began considering the requirements for a new fundamental astronomical reference system (Lieske & Abalakin 1990; Hughes, Smith, & Kaplan 1991). The resulting series of IAU resolutions, passed in 1991, 1994, 1997, and 2000 effectively form the specifications for the ICRS. The axes of the ICRS are defined by the adopted positions of a specific set of extragalactic objects, which are assumed to have no measurable proper motions. The ICRS axes are consistent, to about 0.02 arcsecond, with the equator and equinox of J2000.0 defined by the dynamics of the Earth. However, the ICRS axes are meant to be regarded as fixed directions in space that have an existence independent of the dynamics of the Earth or the particular set of objects used to define them at any given time.

Feissel & Mignard (1998) have written a concise review of the ICRS adoption and its implications. Seidelmann & Kovalevsky (2002) published a broader review of the ICRS and the new IAU Earth orientation models.

The promotion, maintenance, extension, and use of the ICRS are the responsibilities of IAU Division 1 (Fundamental Astronomy).

3.4 ICRS Implementation

3.4.1 The Defining Extragalactic Frame

The *International Celestial Reference Frame (ICRF)* is a catalog of adopted positions of 608 extragalactic radio sources observed with VLBI, all strong (>0.1 Jy) at S and X bands (wavelengths 13 and 3.6 cm) (Ma & Feissel 1997; URL 10). Most have faint optical counterparts (typically $m_V \gg 18$) and the majority are quasars. Of these objects, 212 are *defining sources* that establish the orientation of the ICRS axes, with origin at the solar system barycenter. Typical position uncertainties for the defining sources are of order 0.5 milliarcsecond; the orientation of the axes is defined from the ensemble to an accuracy of about 0.02 milliarcsecond. As described in section 3.4.4, these axes correspond closely to what would conventionally be described as "the mean equator and equinox of J2000.0".

The International Earth Rotation and Reference Systems Service (IERS) monitors the radio sources involved in the ICRF. This monitoring is necessary because, at some level, most of the sources are variable in both flux and structure and the centers of emission can display spurious motions. It is possible that, eventually, the defining list of sources will have to be amended to maintain the fixed orientation of the overall frame.

3.4.2 The Frame at Optical Wavelengths

The ICRS is realized at optical wavelengths by stars in the Hipparcos Catalogue of 118,218 stars, some as faint as visual magnitude 12 (ESA 1997). Only stars with uncomplicated and well-determined proper motions (e.g., no known binaries) are used for the ICRS realization. This subset, referred to as the Hipparcos Celestial Reference Frame (HCRF), comprises 85% of the stars in the Hipparcos catalog. Hipparcos star coordinates and proper motions are given within the ICRS (J2000.0) coordinate system but are listed for epoch J1991.25. (That is, the catalog effectively represents a snapshot of the positions and motions of the stars taken on 2 April 1991, a date that is near the mean epoch of the Hipparcos observations.) At the catalog epoch, Hipparcos uncertainties for stars brighter than 9th magnitude have median values somewhat better than 1 milliarcsecond in position and 1 milliarcsecond/year in proper motion (ESA 1997; Mignard 1997). Thus, projected to epoch J2000.0, typical Hipparcos star position errors are in the range 5–10 milliarcseconds.

3.4.3 Standard Algorithms

Chapters 1, 2, 5, and 6 of this circular describe IAU-sanctioned algorithms used in the construction, maintenance, and use of the ICRS.

The 2000 IAU resolutions provide the relativistic metric tensors for what it called the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS), as well as expressions for the transformation between the two systems; see Chapter 1 and res. B1.3 of 2000. As noted in Chapter 1, the resolutions specify only the relativistic basis of the two reference systems, and there is no prescription given for establishing the axis directions. The BCRS and GCRS could therefore be considered families of coordinate systems, each member differing from the others only in overall orientation. The construction of the ICRS (in particular, the analysis of VLBI observations) was consistent with the definition of the BCRS in the resolutions. Thus, the ICRS could be considered one implementation of a BCRS; i.e., a member of the BCRS family. Recently, the IAU Working Group on Nomenclature for Fundamental Astronomy has recommended that the orientation of the BCRS axes be understood to be that of the ICRS/ICRF.

In 2000, the IAU also adopted new models for the computation of the Earth's instantaneous orientation, which affect the analysis of VLBI observations that are used to define and maintain the ICRS, as well as the calculation of various observable quantities from ICRS-compatible reference data. The new models include what is referred to as the *IAU 2000A precession-nutation model*, a new definition of the celestial pole, and two new reference points in the equatorial plane for measuring the rotational angle of the Earth around its instantaneous axis. Despite the IAU action in 2000, some aspects of the models were not finalized until late 2002 (mid 2005 for agreement on the final precession expressions). These algorithms are described in Chapters 5 and 6 of this circular and in the IERS Conventions (2003).

The new Earth orientation models are, of course, relevant only to fundamental observations made from the surface of the Earth. Astrometric observations taken from space platforms, or those that are differential in nature (based on reference objects all located within a small field), do not use these models. There are, of course, other effects that must be taken into account in analyzing astrometric observations — e.g., proper motion, parallax, aberration, and gravitational light-bending — and algorithms for these may be found in Volumes 1 and 3 of the Hipparcos Catalogue documentation (ESA 1997). For analysis of very high accuracy satellite observations, see the development by Klioner (2003).

As described in the Introduction, there are two collections of general-purpose computer subroutines that implement the new IAU-sanctioned algorithms for practical applications: the Standards of Fundamental Astronomy (SOFA), at URL 7, and the Naval Observatory Vector Astrometry Subroutines (NOVAS), at URL 8. NOVAS also implements many of the Hipparcos algorithms, or the equivalent.

For ground-based applications requiring accuracies of no better than 50 milliarcseconds between about 1990 and 2010, the algorithms described in Chapter 3 of the Explanatory Supplement (1992) can still be used with ICRS data. (For such purposes, ICRS data can be treated as being on the dynamical equator and equinox of J2000.0.) A major revision of the Explanatory Supplement to reflect the adoption of the ICRS and all the new models is in progress.

3.4.4 Relationship to Other Systems

The orientation of the ICRS axes is consistent with the equator and equinox of J2000.0 represented by the FK5, within the errors of the latter. See Feissel & Mignard (1998) for a short discussion. Systematically, the FK5 equator is tilted by 22 mas and its origin of right ascension is offset by 23 mas with respect to the ICRS. But the uncertainties of the FK5 equator and right ascension system with respect to the dynamical equator and equinox of J2000.0 are 50 and 80 mas, respectively. Since, at J2000.0, the errors of the FK5 are significantly worse than those of Hipparcos, the ICRS (as realized by the HCRF) can be considered to be a refinement of the FK5 system (ESA 1997) at (or near) that epoch.

The ICRS can also be considered to be a good approximation (at least as good as the FK5) to the conventionally defined dynamical equator and equinox of J2000.0 (Feissel & Mignard 1998), if the latter system is considered to be barycentric. This follows from an IAU resolution passed in 1991 that provided the original specifications for the new fundamental astronomical reference system based on extragalactic objects — what became the ICRS. In fact, the equator is well determined fundamentally from the VLBI observations that are the basis for the ICRS, and the ICRS pole is within 20 milliarcseconds of the dynamical pole.[1] The zero point of VLBI-derived right ascensions is arbitrary, but traditionally has been set by assigning to the right ascension of source 3C 273B a value derived from lunar occultation timings — the Moon's ephemeris thus providing an indirect link to the dynamical equinox. The ICRS origin of right ascension was made to be consistent with that in a group of VLBI catalogs previously used by the IERS, each of which had been individually aligned to the lunar occultation right ascension of 3C 273B. The difference between the ICRS origin of right ascension and the dynamical equinox has been independently measured by two groups that used different definitions of the equinox, but in both cases the difference found was less than 0.1 arcsecond.

Because of its consistency with previous reference systems, implementation of the ICRS will be transparent to any applications with accuracy requirements of no better than 0.1 arcseconds near epoch J2000.0. That is, for applications of this accuracy, the distinctions between the ICRS, FK5, and dynamical equator and equinox of J2000.0 are not significant.

3.4.5 Data in the ICRS

Although the ICRF and HCRF are its basic radio and optical realizations, the ICRS is gradually being extended to fainter magnitudes and other wavelengths. Thus, an increasing amount of fundamental astronomical data is being brought within the new system. A number of projects for the densification of the ICRS have been completed or are in progress.

As described above, the ICRF consists of the adopted positions of about 600 extragalactic radio sources, a third of which are defining sources. In its original presentation, the ICRF contained 608 extragalactic radio sources, including 212 defining sources. All observational data were part of a common catalog reduction (Ma & Feissel 1997) and thus the adopted coordinates of all the sources are in the ICRS. Of the 396 non-defining sources, 294 are *candidate sources* that do not meet all of the accuracy and observing history requirements of the defining sources but which may at some later time be added to the defining list. The remaining 102 *other sources* show excess apparent position variation and are of lower astrometric quality. ICRF Extension 2 (ICRF-Ext.2) was issued in 2004 (Fey et al. 2004); the positions of the candidate and other sources were refined and 109 new sources were added. The positions of the defining sources were left unchanged.

The VLBA Calibrator Survey is a list of radio sources, with positions in the ICRS, to be used as calibrators for the Very Long Baseline Array and the Very Large Array. The original list was prepared by Beasley et al. (2002); the list has been extended several times and the current version is known as VCS3 (Petrov et al. 2005). The ICRS is also being established at higher radio frequencies (24 and 43 GHz); see, e.g., Jacobs et al. (2005).

In the optical regime, the Tycho-2 Catalogue (Høg et al. 2000) (which supersedes the original Tycho Catalogue and the ACT Reference Catalog) combines a re-analysis of the Hipparcos star mapper observations with data from 144 ground-based star catalogs. The ground-based catalogs include the Astrographic Catalogue (AC), a large photographic project carried out near the beginning of the 20th century involving 20 observatories worldwide. Tycho-2 contains 2,539,913 stars, to about magnitude 12, and combines the accuracy of the recent Hipparcos position measurements with proper motions derived from a time baseline of almost a century. Proper motion uncertainties are 1-3 milliarcseconds/year. At epoch J2000.0, the Tycho-2 positions of stars brighter than 9th magnitude will typically be in error by 20 milliarcseconds. However, the positional accuracy degrades quite rapidly for magnitudes fainter than 9, so that 12th magnitude stars may be expected to have a median J2000.0 position error exceeding 100 milliarcseconds.

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[1] The reason that the ICRS pole is not perfectly aligned with the dynamical pole is complex. The ICRF was created from almost 20 years of VLBI observations from which a grand solution was made for the directions to the extragalactic radio sources and the changing position of the celestial pole. A specific decision made in that analysis (see Figure 5.2 and footnote on page 37) resulted in an offset of the ICRF (hence ICRS) pole at J2000.0. With respect to the ICRS X and Y axes, the dynamical mean pole has coordinates on the unit sphere, in milliarcseconds, of approximately (-16.6,-6.8).

Also in the optical band, the UCAC catalog is nearing completion and will provide ICRS-compatible positions and proper motions for stars as faint as visual magnitude 16. See Zacharias (2004) for information on the second release of UCAC data.

The ICRS has been extended to the near infrared through the 2MASS survey (Cutri et al. 2003). This ground-based program provides positions for 471 million point sources, most of which are stars, observed in the J, H, and K_s infrared bands.

The Jet Propulsion Laboratory DE405/LE405 planetary and lunar ephemerides (usually just referred to as DE405) (Standish 1998b) have been aligned to the ICRS. These ephemerides provide the positions and velocities of the nine major planets and the Moon with respect to the solar system barycenter, in rectangular coordinates. The data are represented in Chebyshev series form and Fortran subroutines are provided to read and evaluate the series for any date and time. DE405 spans the years 1600 to 2200; a long version, DE406, spans the years −3000 to +3000 with lower precision. See Chapter 4.

The barycentric data tabulated in *The Astronomical Almanac* are in the ICRS beginning with the 2003 edition. Planetary and lunar ephemerides are derived from DE405/LE405. *The Astronomical Almanac* for 2006 is the first edition fully to support the new ICRS-related algorithms, including the new IAU Earth rotation models. Geocentric coordinates are therefore given with respect to the GCRS.

3.5 Formulas

A matrix **B** is required to convert ICRS data to the dynamical mean equator and equinox of J2000.0 (the "J2000.0 system"), the latter considered for this purpose to be a barycentric system. The same matrix is used in the geocentric transformations described in Chapters 5 and 6 to adjust vectors in the GCRS (the "geocentric ICRS") so that they can be operated on by the conventional precession and nutation matrices. The matrix **B** is called the *frame bias matrix*, and it corresponds to a fixed set of very small rotations. In the barycentric case it is used as follows:

$$\mathbf{r}_{\mathrm{mean}(2000)} = \mathbf{B}\, \mathbf{r}_{\mathrm{ICRS}} \tag{3.1}$$

where $\mathbf{r}_{\mathrm{ICRS}}$ is a vector with respect to the ICRS and $\mathbf{r}_{\mathrm{mean}(2000)}$ is a vector with respect to the dynamical mean equator and equinox of J2000.0. Both of the **r**'s are column vectors and, if they represent a direction on the sky, are of the general form

$$\mathbf{r} = \begin{pmatrix} \cos\delta \, \cos\alpha \\ \cos\delta \, \sin\alpha \\ \sin\delta \end{pmatrix} \tag{3.2}$$

where α is the right ascension and δ is the declination, with respect to the ICRS or the dynamical system of J2000.0, as appropriate.

In the geocentric case, $\mathbf{r}_{\mathrm{ICRS}}$ is replaced by $\mathbf{r}_{\mathrm{GCRS}}$ and $\mathbf{r}_{\mathrm{mean}(2000)}$ is then a geocentric vector. This transformation must be carried out, for example, before precession is applied to GCRS vectors, since the precession algorithm assumes a dynamical coordinate system. That is, the above transformation is a necessary step in obtaining coordinates with respect to the mean equator and equinox of date, if one starts with ICRS reference data. See Chapter 5 for more information.

The matrix **B** is, to first order,

$$\mathbf{B} = \begin{pmatrix} 1 & d\alpha_0 & -\xi_0 \\ -d\alpha_0 & 1 & -\eta_0 \\ \xi_0 & \eta_0 & 1 \end{pmatrix} \tag{3.3}$$

where $d\alpha_0 = -14.6$ mas, $\xi_0 = -16.6170$ mas, and $\eta_0 = -6.8192$ mas, all converted to radians (divide by 206 264 806.247). The values of the three small angular offsets are taken from the IERS Conventions (2003). They can be considered adopted values; previous investigations of the dynamical-ICRS relationship obtained results that differ at the mas level or more, depending on the technique and assumptions. See the discussion in Hilton & Hohenkerk (2004). The angles ξ_0 and η_0 are the ICRS pole offsets, and $d\alpha_0$ is the offset in the ICRS right ascension origin with respect to the dynamical equinox of J2000.0, as measured in an inertial (non-rotating) system.

The above matrix can also be used to transform vectors from the ICRS to the FK5 system at J2000.0. Simply substitute $d\alpha_0 = -22.9$ mas, $\xi_0 = 9.1$ mas, and $\eta_0 = -19.9$ mas. However, there is also a time-dependent rotation of the FK5 with respect to the ICRS (i.e., a slow spin), reflecting the non-inertiality of the FK5 proper motions; see Mignard & Frœschlé (2000).

Although the above matrix is adequate for most applications, a more precise result can be obtained by using the second-order version:

$$\mathbf{B} = \begin{pmatrix} 1 - \frac{1}{2}(d\alpha_0^2 + \xi_0^2) & d\alpha_0 & -\xi_0 \\ -d\alpha_0 - \eta_0\xi_0 & 1 - \frac{1}{2}(d\alpha_0^2 + \eta_0^2) & -\eta_0 \\ \xi_0 - \eta_0 d\alpha_0 & \eta_0 + \xi_0 d\alpha_0 & 1 - \frac{1}{2}(\eta_0^2 + \xi_0^2) \end{pmatrix} \tag{3.4}$$

The above matrix, from Slabinski (2005), is an excellent approximation to the set of rotations $\mathbf{R}_1(-\eta_0)\mathbf{R}_2(\xi_0)\mathbf{R}_3(d\alpha_0)$, where $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ are standard rotations about the x, y, and z axes, respectively (see "Abbreviations and Symbols Frequently Used" for precise definitions).

Chapter 4

Ephemerides of the Major Solar System Bodies

Relevant IAU resolutions: (none)

Summary The de facto standard source of accurate data on the positions and motions of the major solar system bodies is currently the ephemeris designated DE405/LE405 (or simply DE405) developed at the Jet Propulsion Laboratory. This ephemeris provides instantaneous position and velocity vectors of the nine major planets and the Earth's Moon, with respect to the solar system barycenter, for any date and time between 1600 and 2201. Lunar rotation angles are also provided. The ephemeris has been the basis for the tabulations in *The Astronomical Almanac* since the 2003 edition. The DE405 coordinate system has been aligned to the ICRS.

IAU-standard data on the sizes, shapes, rotational parameters, and latitude-longitude systems for the major planets and their satellites are given in the reports of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements, issued every three years.

4.1 The JPL Ephemerides

A list of positions of one or more solar system bodies as a function of time is called an *ephemeris* (pl. *ephemerides*). An ephemeris can take many forms, including a printed tabulation, a sequential computer file, or a piece of software that, when interrogated, computes the requested data from series approximations or other mathematical schemes.

Ephemerides of the major solar system bodies, with respect to the solar system barycenter, have been calculated for many years at the Jet Propulsion Laboratory (JPL) to support various spacecraft missions. These ephemerides have been widely distributed and, because of their quality, have become the de facto standard source of such data for applications requiring the highest accuracy. Between the early 1980s and about 2000, the JPL ephemeris designated DE200/LE200 was most frequently used for such applications; it was the basis for the tabulations in *The Astronomical Almanac* from the 1984 to 2002 editions. A more recent JPL ephemeris, DE405/LE405, has now come into widespread use, and has been the basis for *The Astronomical Almanac* since the 2003 edition. These ephemerides are usually referred to as just DE200 and DE405, respectively. Neither

DE200 nor DE405 have been the subject of any IAU resolution, although they have been frequently reported on at various IAU-sponsored meetings, and DE405 is a recommended standard of the IERS (IERS Conventions 2003). A comparison of DE405 with DE200, with an estimate of their errors, has been given by Standish (2004).

The JPL ephemerides are computed by an N-body numerical integration, carried out in a barycentric reference system which is consistent, except for the time scale used, with the Barycentric Celestial Reference System (BCRS) described in Chapter 1. The equations of motion, the method of integration, and the techniques used to adjust the starting conditions of the integration so that the results are an optimal fit to observations are described in Chapter 5 of the Explanatory Supplement (1992). That chapter specifically describes DE200, but the basic procedures are largely the same for all of the JPL ephemerides.

The position and velocity data provided by the JPL ephemerides represent the centers of mass of each planet-satellite system (although data for the Earth and Moon can be extracted separately). Therefore, the positions, when converted to geocentric apparent places — angular coordinates as seen from Earth — do not precisely indicate the center of the apparent planetary disk. Displacements can amount to a few tens of milliarcseconds for Jupiter and Saturn, a few milliarcseconds for Uranus and Neptune, and about 0.1 arcsecond for Pluto.

4.2 DE405

The JPL DE405/LE405 ephemeris provides the coordinates and velocities of the major planets, the Sun, and the Earth's Moon for the period 1600 to 2200 (Standish 1998b). The position and velocity 3-vectors are in equatorial rectangular coordinates referred to the solar system barycenter. The reference frame for the DE405 is the ICRF; the alignment onto this frame, and therefore onto the ICRS, has an estimated accuracy of a few milliarcseconds, at least for the inner-planet data. Optical, radar, laser, and spacecraft observations were analyzed to determine starting conditions for the numerical integration and values of fundamental constants such as the Earth/Moon mass ratio and the length of the astronomical unit in meters. In addition to the planetary and lunar coordinates and velocities, the ephemerides, as distributed, include the nutation angles of the Earth and the rotation (libration) angles of the Moon. (Note, however, that the nutation angles are not derived from the IAU 2000A theory described in Chapter 5.)

As described in Chapter 2, DE405 was developed in a barycentric reference system using $\mathrm{T_{eph}}$, a barycentric coordinate time (Standish 1998a). $\mathrm{T_{eph}}$ is rigorously equivalent to TCB in a mathematical sense, differing only in rate: the rate of $\mathrm{T_{eph}}$ matches the average rate of TT, while the rate of TCB is defined by the SI system. The IAU time scale TDB, often (but erroneously) considered to be the same as $\mathrm{T_{eph}}$, is a quantity that cannot be physically realized, due to its flawed definition. So, in fact, the use of the name TDB actually refers to quantities based on or created with $\mathrm{T_{eph}}$.[1] Astronomical constants obtained from ephemerides based on $\mathrm{T_{eph}}$ (or TDB) are not in the SI system of units and must therefore be scaled for use with TCB or other SI-based time scales.

The ephemerides are distributed by JPL as plain-text (ASCII) computer files of Chebyshev series coefficients and Fortran source code. Third-party C versions of the code are also available and, for Unix users, the data files can be downloaded in binary form. Once the system is installed on a given computer, a Fortran subroutine named PLEPH can be called to provide the position and velocity of any requested body, at any date and time; PLEPH supervises the process of reading the Chebyshev file and evaluating the appropriate series. Normally the position and velocity vectors returned are expressed in units of AU and AU/day, respectively. The date/time requested must be expressed as a $T_{\rm eph}$ or TDB Julian date. (An entry named DPLEPH is provided that allows the input Julian date to be split into two parts for greater precision.) And, since $|TT - T_{\rm eph}| < 0.002$ s, a TT Julian date may be used for applications not requiring the highest accuracy. The data and software files can be obtained on CD-ROM from Willmann-Bell, Inc. (URL 11a), or downloaded from a JPL ftp server (URL 11b). A "README" file provides export information and software documentation (available separately at URL 12).

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[1] Because of this, the IAU Working Group on Nomenclature for Fundamental Astronomy has recommended changing the definition of TDB to be consistent with that of $\mathrm{T_{eph}}$.

An extended version of DE405/LE405, called DE406/LE406, is available that spans the years –3000 to +3000, but with coordinates given to lower precision (they are represented by shorter Chebyshev series). The nutation angles and the lunar rotation angles are also omitted from the DE406 files. DE406 is provided only in Unix binary format. These files are about 1/3 the size of those for DE405 for a given span of time. The additional error in the coordinates (DE406 – DE405) may amount to 25 m for the planets and 1 m for the Moon, which may be significant for some applications.

The NOVAS software package mentioned in the Introduction provides an interface to an existing DE405 or DE406 installation through Fortran subroutine SOLSYS or C function *ephemeris.*

4.3 Sizes, Shapes, and Rotational Data

The IAU/IAG[2] Working Group on Cartographic Coordinates and Rotational Elements (URL 13) produces a report every three years (for each IAU General Assembly) giving the best estimates for the dimensions and rotational parameters of the planets, satellites, and asteroids, as far as is known. The working group is also responsible for establishing latitude-longitude coordinate systems for these bodies. The rotational elements given in the report for the 2000 General Assembly (Seidelmann et al. 2002) serve to orient these coordinate systems within the ICRS as a function of time. (Note that the time scale in this report is TDB, not TCB as stated.) The working group's reports are the basis for the physical ephemerides of the planets given in *The Astronomical Almanac.*

Although the rotational elements of the Earth and Moon are given in each report for completeness, the expressions given there provide only an approximation to the known motions and should not be used for precise work (e.g., for the Earth, precession is accounted for only to first order and nutation is neglected). Lunar rotation (libration) angles can be obtained from DE405, and Chapters 5 and 6 of this circular describe algorithms for the precise instantaneous alignment of the terrestrial coordinate system within the GCRS (the "geocentric ICRS").

4.4 DE405 Constants

The constants below were used in or determined from the DE405 ephemeris. Many DE405 constants are expressed in what have been called "TDB units", rather than SI units. (Perhaps it is more proper to say that the constants have TDB-compatible values.) A scaling factor, $K = 1/(1 - L_B)$, where L_B is given below, is required to convert such constants, with dimensions of length or time, to SI units (Irwin & Fukushima 1999). Dimensionless quantities such as mass ratios do not require scaling.

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[2] IAG = International Association of Geodesy

$$L_B = 1.550\,519\,767\,72\times10^{-8} \implies K = 1.000\,000\,015\,505\,198 \tag{4.1}$$

The value of L_B is taken from note 3 of res. B1.5 of 2000.

These conversions also depend on assumptions about what constants are truly fundamental and what their values are (Klioner 2005); therefore, the SI value of the astronomical unit in meters is not given below. The planetary masses below include contributions from satellites and atmospheres.

Constant	**Symbol**	**DE405 Value**	**Equivalent SI value**
astronomical unit in seconds	τ_A	499.004 783 806 1 s	499.004 786 385 2 s
astronomical unit in meters	A or $c\tau_A$	149 597 870 691 m	
heliocentric gravitational constant	GS or $GM_\odot$	$1.327\,124\,400\,2\times10^{20}$ $\mathrm{m^3s^{-2}}$	$1.327\,124\,420\,8\times10^{20}$ $\mathrm{m^3s^{-2}}$
Moon/Earth mass ratio	μ	1/81.30056	
Sun/planet mass ratios:			
Mercury		6 023 600	
Venus		408 523.71	
Earth + Moon		328 900.561400	
Earth		332 946.050895…	
Mars		3 098 708	
Jupiter		1 047.3486	
Saturn		3 497.898	
Uranus		22 902.98	
Neptune		19 412.24	
Pluto		135 200 000	

Chapter 5

Precession and Nutation

Relevant IAU resolutions: B1.6, B1.7 of 2000

Summary Precession and nutation are really two aspects of a single phenomenon, the overall response of the spinning, oblate, elastic Earth to external gravitational torques from the Moon, Sun, and planets. As a result of these torques, the orientation of the Earth's rotation axis is constantly changing with respect to a space-fixed (locally inertial) reference system. The motion of the celestial pole among the stars is conventionally described as consisting of a smooth long-term motion called precession upon which is superimposed a series of small periodic components called nutation.

The algorithms for precession used generally from about 1980 through 2000 (in *The Astronomical Almanac* from the 1984 through 2005 editions) were based on the IAU (1976) value for the rate of general precession in ecliptic longitude (5029.0966 arcseconds per Julian century at J2000.0). Nutation over most of the same time period was given by the 1980 IAU Theory of Nutation. However, not long after these algorithms were widely adopted, it became clear that the IAU (1976) rate of precession had been overestimated by approximately 3 milliarcseconds per year. Further observations also revealed periodic errors of a few milliarcseconds in the 1980 IAU Theory of Nutation. For many applications these errors are negligible, but they are significant at the level of the best ground-based astrometry and geodesy.

As part of the 2000 IAU resolutions, the IAU 2000A precession-nutation model was introduced, based on an updated value for the rate of precession and a completely new nutation theory. As before, the model actually consists of two parts, a precession algorithm describing the smooth secular motion of the celestial pole and a nutation algorithm describing the small periodic variations in the pole's position. The precession algorithm consists of short polynomial series for the values of certain angles. The sines and cosines of these angles, in combination, then define the elements of a precession matrix, **P**. The nutation algorithm consists of a rather long series expansion in Fourier terms for the angular offsets, in ecliptic longitude and latitude, of the actual celestial pole (as modeled) from the precession-only pole (true pole – mean pole). The sines and cosines of these offsets, in combination, then define the elements of a nutation matrix, **N**. The **P** and **N** matrices are applied to the coordinates of celestial objects, expressed as 3-vectors, to transform them from the equator and equinox of one epoch to the equator and equinox of another.

5.1 Aspects of Earth Rotation

The Earth is a relatively well-behaved rotating body, and illustrates the three basic elements of classical spin dynamics: precession, nutation, and Eulerian wobble. In fact, to first order, the Earth can be considered to be a rigid "fast top", and very good approximations to its rotational motion can be obtained from elementary developments. Although the effects of the Earth's liquid core, elastic mantle, and oceans are not negligible for modern observations, they can be considered to be small perturbations on the rigid-body motion. Since the Earth is nearly spherical and experiences relatively weak torques, its axis of rotation moves slowly with respect to both the stars and the body of the Earth itself.

The orientation of any rotating rigid body can be described as a time series of three Euler angles that relate a body-fixed coordinate system to a space-fixed coordinate system. If the body-fixed coordinate system can be defined such that the rate of change of one of the three Euler angles is much greater than that of the other two — as is the case for the Earth — then the rotational kinematics are usually described in terms of the slowly changing orientation of an axis of rotation passing through the body's center of mass. We can equivalently speak of the kinematics of the pole: one of the points where the axis of rotation intersects the body's surface or, extended to infinity, the "celestial sphere". For this kinematic construction to work well, the angular motion of the axis or pole should be small and nearly linear over one rotation, predictable from theory, and observable.

However, as was pointed out by Eubanks (1993), when we use such an axis or pole, we need five angles, not three, to fully describe the instantaneous orientation of the body: two angles to describe the orientation of the body with respect to the axis, one to describe the angle of the body's rotation about the axis, and two more to describe the orientation of the axis in the fixed external system ("inertial space"). For the Earth, these five angles correspond to the five standard parameters of Earth orientation disseminated by organizations such as the IERS: the coordinates of the pole, x_p and y_p, measured in a terrestrial coordinate system; the Universal Time difference, UT1–UTC; and the celestial pole offsets, $d\psi$ and $d\epsilon$, measured in a celestial coordinate system. Phenomenologically, the parameters divide up as follows: x_p and y_p describe polar motion, the variations in the position of the pole with respect to the Earth's crust; UT1–UTC measures the integrated variation in length of day, the departure from a constant angular rate of rotation; and $d\psi$ and $d\epsilon$ are the errors in the computed position of the celestial pole, reflecting deficiencies in the adopted algorithms for precession and nutation.

What we call polar motion corresponds, in rigid-body rotation, to the free Eulerian wobble of the figure axis about the rotation axis. On the real Earth, the phenomenon is not that simple. From an Earth-fixed (rotating) frame of reference, polar motion is a 10-meter (0.3 arcsecond) quasi-circular excursion in the pole position, with principal periods of 12 and 14 months. The 14-month component corresponds to the Eulerian wobble, as modified by the Earth's elasticity, while the 12-month component undoubtedly is a seasonal effect. Smaller, quasi-random variations are not well understood. None of the components is regular enough to permit reliable predictions, and polar motion must be obtained from observations.

Variations in the Earth's rotation rate are due to several causes. There are fortnightly, monthly, semiannual, and annual tidal effects, and other short-term and seasonal changes are largely due to exchange of angular momentum with the atmosphere. Longer-term variations (decade fluctuations) are less well understood. For a discussion of time scales that are based on the variable rotation of the Earth, see sections 2.1 and 2.3.

Precession and nutation refer to the changing orientation of the Earth's axis, with respect to a space-fixed (kinematically non-rotating) system, in response to external torques. The torques are due to the gravitational attraction of the Moon and Sun (and, to a much lesser extent, the planets) on the equatorial bulge of the Earth. Precession and nutation are really different aspects of a single physical phenomenon, and it has become more common in recent years to write "precession-nutation". Precession is simply the secular term in the response, while nutation is the set of periodic terms. On the celestial sphere, the celestial pole traces out a circle, about 23° in radius, centered on the ecliptic pole (the direction orthogonal to the ecliptic plane), taking about 26,000 years to complete one circuit (≈20 arcseconds/year). Precession theory describes this smooth, long-term motion, and the precessional pole is referred to as the *mean pole* (the orthogonal plane is the *mean equator*). But the pole also undergoes a hierarchy of small epicyclic motions, the largest of which is a 14×18 arcsecond ellipse traced out every 18.6 years (see Fig. 1). Nutation theory describes these periodic motions. To get the path of the *true pole* on the celestial sphere (i.e., the direction of the Earth's axis in space), it is necessary to compute both precession and nutation; conventionally, they are described by separate time-dependent rotation matrices, $\mathbf{P}(t)$ and $\mathbf{N}(t)$, which are either multiplied together or applied sequentially.

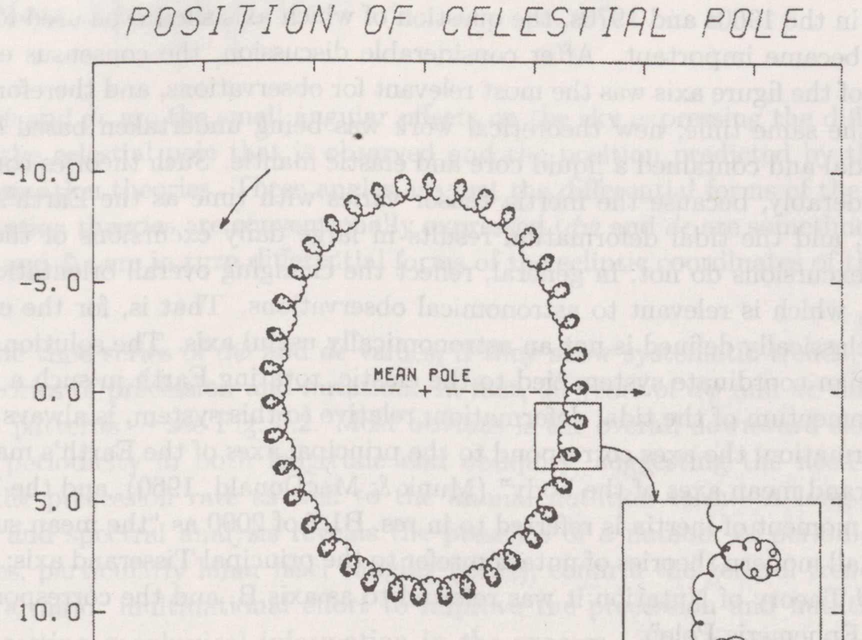

Figure 5.1 The path of the true celestial pole on the sky, over an 18-year period, compared to the mean pole. The mean pole moves along a smooth arc at a rate of 20 arcseconds per year due to precession only. The complex epicyclic motion of the true pole is nutation. The inset shows the detail of one year's motion.

5.2 Which Pole?

In theoretical developments of Earth rotation, the first issue that must be confronted is the definition of the celestial pole. If the Earth were a rigid oblate spheroid, there would be three possible axes, and corresponding poles, to choose from: the angular momentum axis; the rotation axis, defined by the instantaneous angular velocity vector; and the figure axis, which is the body-fixed axis orthogonal to the geometric equator and along the unique eigenvector of the Earth's inertia tensor. The distinctions among the axes arise from the physics of rotation. For example, as previously noted, in a rotating rigid body, the free Eulerian wobble describes the motion of the figure axis with respect to the rotation axis (or vice versa). The analog of this on the real Earth is polar motion.

In the previous section, precession-nutation was described as the changing orientation of the Earth's axis in response to external torques, expressed in a space-fixed (non-rotating) frame of reference. Which axis? The principal components of the response are rather large, amounting to many arcseconds over the course of a year, and are nearly the same for all three axes. (However, the three axes cannot coincide in the presence of external torques.) For a rigid Earth, the forced oscillations of the figure and rotation axes differ by about 10 milliarcseconds, and those of the angular momentum and rotation axes differ by only about 1 milliarcsecond. Until the mid-20th century, observations were not accurate enough to distinguish between the axes, so the choice of the best axis for theory was academic. But with improving observational accuracy and new techniques coming online in the 1960s and 1970s, the question of which axis should be used for the theoretical developments became important. After considerable discussion, the consensus emerged that the forced motion of the figure axis was the most relevant for observations, and therefore also for theory.

At about the same time, new theoretical work was being undertaken based on Earth models that were triaxial and contained a liquid core and elastic mantle. Such theories complicate the axis question considerably, because the inertia tensor varies with time as the Earth's shape responds to tidal forces, and the tidal deformation results in large daily excursions of the Earth's axis of figure. These excursions do not, in general, reflect the changing overall orientation of the Earth's crust in space, which is relevant to astronomical observations. That is, for the elastic Earth, the figure axis as classically defined is not an astronomically useful axis. The solution is to construct a rotating cartesian coordinate system tied to the elastic, rotating Earth in such a way that (1) the net angular momentum of the tidal deformation, relative to this system, is always zero; and (2) for zero tidal deformation, the axes correspond to the principal axes of the Earth's mantle. These axes are the "Tisserand mean axes of the body" (Munk & MacDonald 1960), and the Tisserand axis of the maximum moment of inertia is referred to in res. B1.7 of 2000 as "the mean surface geographic axis". Almost all modern theories of nutation refer to the principal Tisserand axis; in the previously used 1980 IAU Theory of Nutation it was referred to as axis B, and the corresponding pole called the "Celestial Ephemeris Pole".

However, even if we have chosen an axis that best reflects the overall rotation of stations (observatories) on the Earth's surface, a further complication arises as the observations and theoretical developments become more sensitive to short-period motions. The problem is the small but non-negligible circular components of nutation or polar motion with periods near one day. One can imagine the geometric confusion that arises when the pole undergoes a circular motion in one rotation period; in fact, it becomes difficult to disentangle the various effects, and our conventional labels become nearly meaningless. For example, any prograde nearly-diurnal nutation is equivalent to a long-period variation in polar motion, and any retrograde nearly-diurnal polar motion appears as a long-period nutation component (Capitaine 2000). In practice, this means a potential "leakage" or "crossover" from the Earth orientation parameters x_p and y_p to $d\psi$ and $d\epsilon$ or vice versa. The only practical solution is an explicit (although somewhat arbitrary) cutoff in the periods of what is considered precession-nutation, embodied in the definition of the celestial pole.

Therefore, the new IAU definition of the celestial pole to be used for the new precession-nutation model (res. B1.7 of 2000) is defined by the motions of Tisserand mean axis of the Earth with periods greater than two days in the celestial reference system. This pole is called the *Celestial Intermediate Pole* (CIP). The position of the CIP is given by the adopted precession-nutation model plus observational corrections. The word *intermediate* reminds us that the definition of the pole is merely a convention, serving to impose a division between what we call precession-nutation (the Earth orientation angles measured in the celestial system) and polar motion (the Earth orientation angles measured in the terrestrial system). The CIP is the true pole, orthogonal to the true equator of date. Its motion is defined within the Geocentric Celestial Reference System (GCRS) — see Chapter 1. Therefore, the geometric transformations described in this chapter (as well as those in Chapter 6) all apply within a geocentric system. The GCRS can be described loosely as the "geocentric ICRS", since its axis directions are obtained from those of the ICRS.

5.3 The New Models

The variables $d\psi$ and $d\epsilon$ are the small angular offsets on the sky expressing the difference between the position of the celestial pole that is observed and the position predicted by the conventional precession and nutation theories. These angles are just the differential forms of the angles $\Delta\psi$ and $\Delta\epsilon$ in which nutation theories are conventionally expressed ($d\psi$ and $d\epsilon$ are sometimes labeled $\Delta\Delta\psi$ and $\Delta\Delta\epsilon$). $\Delta\psi$ and $\Delta\epsilon$ are in turn differential forms of the ecliptic coordinates of the celestial pole (see Fig. 5.1).

Obviously the time series of $d\psi$ and $d\epsilon$ values, if they show systematic trends, can be used to improve the theories of precession and nutation. In fact, 20 years of $d\psi$ and $d\epsilon$ values from VLBI show significant patterns — see Fig. 5.2. Most obvious is the overall downward slope in longitude and an annual periodicity in both longitude and obliquity, suggesting the need for substantial corrections to the precession rate as well to the annual nutation term. A long-period sinusoid is also evident, and spectral analysis reveals the presence of a number of periodic components.[1] Other techniques, particularly lunar laser ranging (LLR), confirm the general trends. As a result, there has been a major multinational effort to improve the precession and nutation formulation and obtain interesting geophysical information in the process. This project, coordinated by an IAU/IUGG[2] working group, has involved dozens of investigators in several fields, and the resulting algorithms, taken together, are referred to as the IAU 2000A precession-nutation model.

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[1] The figure indicates the origin of the ICRS "frame bias" discussed in Chapter 3. The pole offsets shown are taken from the solution for the ICRF catalog. The ICRS frame biases in longitude and obliquity are essentially the values, at J2000.0 (Time=100), of the two curves fitted to the data. The data was arbitrarily zeroed near the beginning of the data span, which led to non-zero values at J2000.0.

[2] IUGG = International Union of Geodesy and Geophysics

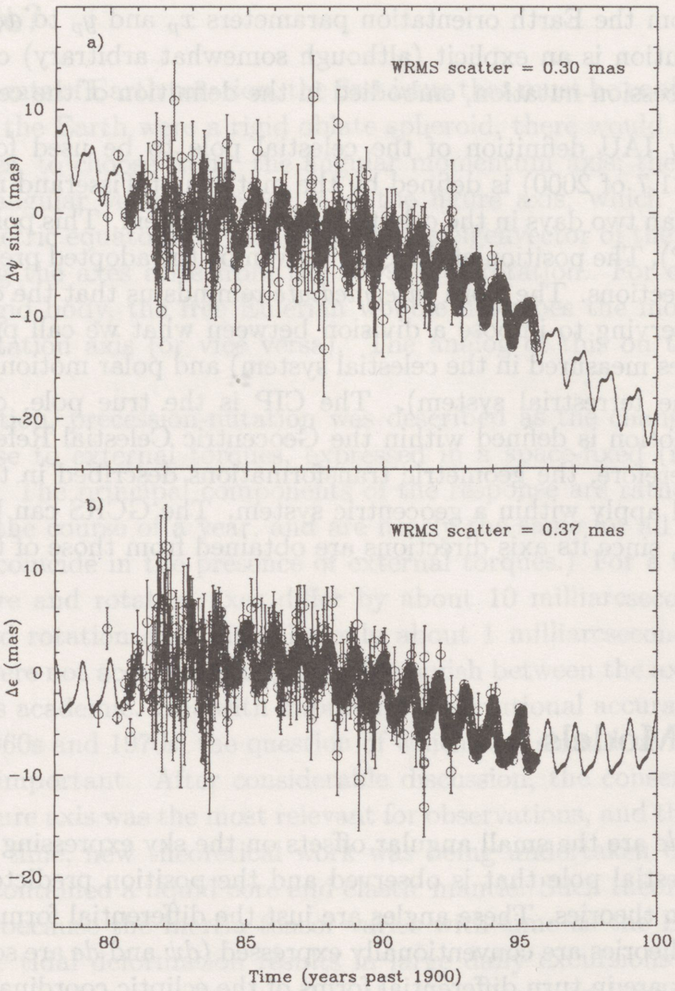

Figure 5.2 Observed values of celestial pole offsets from VLBI data. Offsets in longitude have been multiplied by the sine of the obliquity to allow the same scale to be used for both components. Circled points with error bars represent the offset of the observed pole with respect to the computed pole, and the solid line in each plot is a curve fitted to the data. The computed pole is given by the Lieske et al. (1977) precession expressions and the 1980 IAU Theory of Nutation. These plots are from Ma et al. (1998).

The VLBI observations of $d\psi$ and $d\epsilon$ indicate the error in the computed position of the pole with respect to a space-fixed system defined by the positions of extragalactic objects. However, the conventional expressions for precession and nutation have used angles measured with respect to the ecliptic, a plane to which VLBI is not sensitive. The ecliptic plane has a slow precessional movement of its own due to planetary perturbations on the heliocentric orbital motion of the Earth-Moon barycenter.[3] In the theoretical developments it is necessary to distinguish between *precession of the equator* and *precession of the ecliptic*, which were formerly called, respectively, *lunisolar precession* and *planetary precession.* Both types of precession are measured with respect to a space-fixed system. The algorithms for precession and nutation provide the motion of the

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[3] The *mean ecliptic* is always implied. This is the smoothly moving plane that does not undergo the periodic oscillations of the instantaneous orbital plane of the Earth.

Table 5.1 Precession-Nutation: Old & New

Values in arcseconds at J2000.0

Quantity	Old value	New value	New–Old
General precession in longitude (/cen)	5029.0966	5028.796195	−0.3004
Mean obliquity	84381.448	84381.406	−0.042
Mean obliquity rate (/cen)	−46.8150	−46.836769	−0.0218
In-phase nutation amplitudes:			
18.6-year longitude	−17.1966	−17.2064161	−0.0098
18.6-year obliquity	9.2025	9.2052331	0.0027
9.3-year longitude	0.2062	0.2074554	0.0013
9.3-year obliquity	−0.0895	−0.0897492	−0.0002
annual longitude	0.1426	0.1475877	0.0050
annual obliquity	0.0054	0.0073871	0.0020
semiannual longitude	−1.3187	−1.3170906	0.0016
semiannual obliquity	0.5736	0.5730336	−0.0006
122-day longitude	−0.0517	−0.0516821	0.0000
122-day obliquity	0.0224	0.0224386	0.0000
monthly longitude	0.0712	0.0711159	−0.0001
monthly obliquity	−0.0007	−0.0006750	0.0000
semimonthly longitude	−0.2274	−0.2276413	−0.0002
semimonthly obliquity	0.0977	0.0978459	0.0001

equator, as appropriate for most observations, but generally use a moving ecliptic as a reference plane for at least some of the angles involved (there are different formulations of precession using different angle sets). This allows the precession and nutation transformations to properly account for the motion of the equinox as well as that of the equator. The precession of the ecliptic is obtained from theory (although indirectly tied to observations through the JPL DE405 ephemeris), as are the high-order (unobserved) components of the precession of the equator. However, because of the mix of theory and observation that is involved in the final expressions, raw corrections to rates of precession from VLBI observations will not in general propagate exactly to the familiar precession quantities.

The changes in the amplitudes of the nutation components are also not directly taken from these observations; instead, a new nutation theory is developed and fit to observations by allowing a small number of geophysical constants to be free parameters. These parameters are constants in a "transfer function" that modifies the amplitudes of the terms from a rigid-Earth nutation development. Since there are fewer solved-for geophysical constants than the number of terms with observed amplitudes, the fit cannot be perfect. For the IAU 2000A model, 7 geophysical parameters were determined based on the observed amplitudes of 21 nutation terms (prograde and retrograde amplitudes for each) together with the apparent change in the rate of precession in longitude. Note that the number of observational constraints and the number of free parameters in the model are both quite small compared to the 1365 terms in the new, full nutation series.

Table 5.1 compares the old and new values, at epoch J2000.0, of some of the primary quantities involved in the precession and nutation algorithms. In the table, all quantities are in arcseconds, and the rates (marked /cen) are per Julian century of TDB (or TT). The longitude components should be multiplied by the sine of the obliquity (≈0.3978) to obtain the corresponding motion of the pole on the celestial sphere. The new mean obliquity at J2000.0 is 23° 26′ 21.″406. The theories from which the values are taken are:

- Old precession: Lieske et al. (1977), based on the IAU (1976) values for general precession and the obliquity at J2000.0, shown in the table
- Old nutation: 1980 IAU Theory of Nutation: Wahr (1981), based on Kinoshita (1977); see report of the IAU working group by Seidelmann (1982)
- New precession: P03 solution in Capitaine et al. (2003); see report of the IAU working group by Hilton et al. (2006)
- New nutation: Mathews et al. (2002) (often referred to as MHB), based on Souchay et al. (1999); series listed at URL 14

The new precession development will probably be formally adopted by the IAU in 2006. The MHB nutation was adopted in res. B1.6 of 2000, even though the theory had not been finalized at the time of the IAU General Assembly of that year. Used together, these two developments yield the computed path of the Celestial Intermediate Pole (CIP) as well as that of the true equinox. The formulas given below are based on these two developments.

5.4 Formulas

In the development below, precession and nutation are represented as 3×3 rotation matrices that operate on column 3-vectors. The latter are position vectors in a specific celestial coordinate system — which must be stated or understood — with components that are cartesian (rectangular) coordinates. They have the general form

$$\mathbf{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} d\cos\delta\cos\alpha \\ d\cos\delta\sin\alpha \\ d\sin\delta \end{pmatrix} \tag{5.1}$$

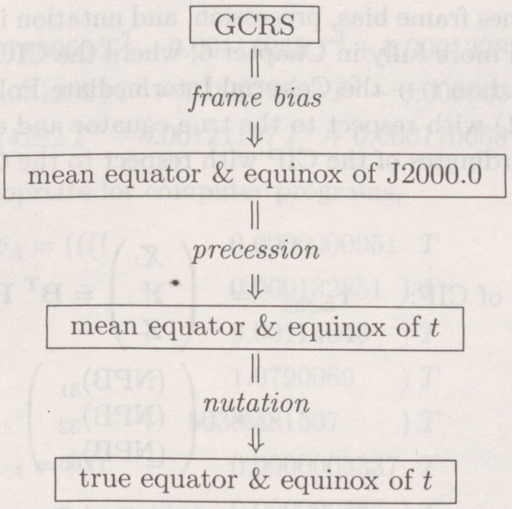
where α is the right ascension, δ is the declination, and d is the distance from the specified origin. For stars and other objects "at infinity" (beyond the solar system), d is often simply set to 1. The celestial coordinate system being used will be indicated by a subscript, e.g., $\mathbf{r}_{\text{GCRS}}$. If we have the vector $\mathbf{r}$ in some coordinate system, then the right ascension and declination in that coordinate system can be obtained from

$$\begin{aligned} \alpha &= \arctan\left(r_y/r_x\right) \\ \delta &= \arctan\left(r_z/\sqrt{r_x^2+r_y^2}\right) \end{aligned} \tag{5.2}$$

where r_x, r_y, and r_z are the three components of $\mathbf{r}$. A two-argument arctangent function (e.g., atan2) will return the correct quadrant for α if r_y and r_x are provided separately.

In the context of traditional equatorial celestial coordinate systems, the adjective *mean* is applied to quantities (pole, equator, equinox, coordinates) affected only by precession, while *true* describes quantities affected by both precession and nutation. This is a computational distinction only, since precession and nutation are simply different aspects of the same physical phenomenon. Thus, it is the *true* quantities that are directly relevant to observations; *mean* quantities now usually represent an intermediate step in the computations, or the final step where only very low accuracy is needed (10 arcseconds or worse) and nutation can be ignored.

Thus, a precession transformation is applied to celestial coordinates to convert them from the mean equator and equinox of J2000.0 to the mean equator and equinox of another date, t. Nutation is applied to the resulting coordinates to transform them to the true equator and equinox of t. These transformations should be understood to be inherently geocentric rotations and they originate in dynamical theories. Generally we will be starting with celestial coordinates in the GCRS, which are obtained from basic ICRS data by applying the usual algorithms for proper place.[4] As discussed in Chapter 3, the ICRS is not based on a dynamically defined equator and equinox and so neither is the GCRS. Therefore, before we apply precession and nutation — and if we require a final accuracy of better than 0.02 arcsecond — we must first apply the frame bias correction (section 3.5) to transform the GCRS coordinates to the dynamical mean equator and equinox of J2000.0. Schematically,


GCRS
frame bias
mean equator & equinox of J2000.0
precession
mean equator & equinox of t
nutation
true equator & equinox of t

Mathematically, this sequence can be represented as follows:

$$\mathbf{r}_{\mathrm{true}(t)} = \mathbf{N}(t)\,\mathbf{P}(t)\,\mathbf{B}\,\mathbf{r}_{\mathrm{GCRS}} \tag{5.3}$$

where $\mathbf{r}_{\mathrm{GCRS}}$ is a direction vector with respect to the GCRS and $\mathbf{r}_{\mathrm{true}(t)}$ is the equivalent vector with respect to the true equator and equinox of t. $\mathbf{N}(t)$ and $\mathbf{P}(t)$ are the nutation and precession rotation matrices, respectively. The remainder of this chapter shows how to compute the elements of these matrices. $\mathbf{B}$ is the (constant) frame-bias matrix given in section 3.5.

The transformation from the mean equator and equinox of J2000.0 to the mean equator and equinox of t is simply

$$\mathbf{r}_{\mathrm{mean}(t)} = \mathbf{P}(t)\,\mathbf{r}_{\mathrm{mean(J2000.0)}} \tag{5.4}$$

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[4] Computing *proper place* involves adjusting the catalog place of a star or other extra-solar system object for proper motion and parallax (where known), gravitational light deflection within the solar system, and aberration due to the Earth's motions. For a solar system object there are comparable adjustments to its position vector taken from a barycentric gravitational ephemeris. See section 1.3. In conventional usage, an *apparent place* can be considered to be a proper place that has been transformed to the true equator and equinox of date. The details of the proper place computations are beyond the scope of this circular but are described in detail in many textbooks on positional astronomy, and in Hohenkerk et al. (1992), Kaplan, et al. (1989), and Klioner (2003).

and the reverse transformation is

$$\mathbf{r}_{\text{mean(J2000.0)}} = \mathbf{P}^{\mathrm{T}}(t)\,\mathbf{r}_{\text{mean}(t)} \tag{5.5}$$

where $\mathbf{P}^{\mathrm{T}}(t)$ is the transpose of $\mathbf{P}(t)$.

What is described above are all conventional, equinox-based transformations. The equinox is the traditional origin of right ascension. An alternative transformation has been developed based on another point on the celestial equator called the Celestial Intermediate Origin (CIO); see res. B1.8 of 2000. The alternative scheme has been introduced because the equinox is based on a barycentric concept (the ecliptic, the Earth's mean orbit) that is not relevant to a geocentric system; furthermore, the equinox suffers from ambiguity of definition below the 0.1 arcsecond level. Additionally, the new scheme cleanly separates different aspects of Earth orientation in the overall transformation between the terrestrial and celestial coordinate systems. The conventional transformations are described in this chapter because of widespread current usage and the fact that even the newest theories of precession and nutation provide the angles needed for these transformations. The new transformation, which combines frame bias, precession, and nutation into a single matrix, is given in section 5.4.3 and is described more fully in Chapter 6, where the CIO is introduced and explained.

The true celestial pole of date t — the Celestial Intermediate Pole (CIP) — has, by definition, unit vector coordinates (0,0,1) with respect to the true equator and equinox of date. Therefore we can obtain the computed coordinates of the CIP with respect to the GCRS by simply reversing the transformation of eq. 5.3:

$$\begin{aligned}\text{Computed position of CIP:}\qquad \mathbf{r}_{\text{GCRS}} &= \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{B}^{\mathrm{T}}\,\mathbf{P}^{\mathrm{T}}(t)\,\mathbf{N}^{\mathrm{T}}(t) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} (\mathrm{NPB})_{31} \\ (\mathrm{NPB})_{32} \\ (\mathrm{NPB})_{33} \end{pmatrix} \qquad\qquad (5.6) \\ \text{where}\qquad \mathbf{NPB} &= \mathbf{N}(t)\,\mathbf{P}(t)\,\mathbf{B}\end{aligned}$$

and where the superscript T's indicate that the transpose of the matrix is used. Daily values of the elements of the combined matrix **NPB** are listed in *The Astronomical Almanac.*

The IERS Conventions (2003) list series expansions that directly provide X and Y, the two most rapidly changing components of the pole position unit vector. Daily values of X and Y are also listed in *The Astronomical Almanac.* The values of X and Y are given in arcseconds and are converted to dimensionless unit vector components simply by dividing them by the number of arcseconds in one radian, 206264.806247.... Also, then, $Z = \sqrt{1 - X^2 - Y^2}$. The values of X and Y are used in the new transformation scheme discussed in section 5.4.3 and in several places in Chapter 6.

5.4.1 Formulas for Precession

To construct the precession matrix for the transformation of coordinates from one date to another, we must evaluate short polynomials for the angles involved. The expressions for these angles in the IAU 2000A model, given below, have only a single time argument, since precession from or to J2000.0 (actually, the TDB equivalent of J2000.0) is assumed. As used in this circular (and *The Astronomical Almanac*), the matrix $\mathbf{P}(t)$ always denotes precession from J2000.0 (TDB) to another date, t. To precess in the opposite direction, the angles are the same but the transpose of the precession matrix, $\mathbf{P}^{\mathrm{T}}(t)$, is used. To precess coordinates from one arbitrary date, t_1, to another, t_2, it is necessary to precess them from t_1 to J2000.0 (using $\mathbf{P}^{\mathrm{T}}(t_1)$), then from J2000.0 to t_2 (using $\mathbf{P}(t_2)$). Where high accuracy is not required, and t_1 and t_2 are not more than a few years apart, a simpler procedure for precession from t_1 to t_2 is available and is given at the end of this subsection.

All expressions given in this subsection are from Section 7 (P03 solution) of Capitaine et al. (2003) and all coefficients are expressed in arcseconds. This is the theory of precession recommended by the IAU Working Group on Precession and the Ecliptic (Hilton et al. 2006).

For a given TDB date and time t, let T be the number of Julian centuries of TDB since 2000 Jan 1, 12^{h} TDB. If the dates and times are expressed as Julian dates, then $T = (t - 2451545.0)/36525$. TT dates and times can be used equally well — the resulting error in precession is only a few $\times 10^{-9}$ arcseconds.

Then the mean obliquity of the ecliptic at J2000.0 (or the equivalent TDB date) is $\epsilon_0 =$ 84381.406 arcseconds and let

$$\begin{aligned}
\psi_A &= 5038.481507\,T - 1.0790069\,T^2 - 0.00114045\,T^3 + 0.000132851\,T^4 - 0.0000000951\,T^5\\
\omega_A &= \epsilon_0 - 0.025754\,T + 0.0512623\,T^2 - 0.00772503\,T^3 - 0.000000467\,T^4 + 0.0000003337\,T^5\\
\chi_A &= 10.556403\,T - 2.3814292\,T^2 - 0.00121197\,T^3 + 0.000170663\,T^4 - 0.0000000560\,T^5
\end{aligned}\tag{5.7}$$

Equivalently, in notation appropriate for computer programs,

$$\begin{array}{rcll}
\psi_A = ((((& - & 0.0000000951 & T\\
 & + & 0.000132851 &)\,T\\
 & - & 0.00114045 &)\,T\\
 & - & 1.0790069 &)\,T\\
 & + & 5038.481507 &)\,T\\
\omega_A = ((((& + & 0.0000003337 & T\\
 & - & 0.000000467 &)\,T\\
 & - & 0.00772503 &)\,T \qquad (5.8)\\
 & + & 0.0512623 &)\,T\\
 & - & 0.025754 &)\,T + \epsilon_0\\
\chi_A = ((((& - & 0.0000000560 & T\\
 & + & 0.000170663 &)\,T\\
 & - & 0.00121197 &)\,T\\
 & - & 2.3814292 &)\,T\\
 & + & 10.556403 &)\,T
\end{array}$$

The precession matrix is then simply $\mathbf{P}(t) = \mathbf{R}_3(\chi_A)\,\mathbf{R}_1(-\omega_A)\,\mathbf{R}_3(-\psi_A)\,\mathbf{R}_1(\epsilon_0)$, where $\mathbf{R}_1$ and $\mathbf{R}_3$ are standard rotations about the x and z axes, respectively (see "Abbreviations and Symbols Frequently Used" for precise definitions). This 4-angle precession formulation is comprised of

1. A rotation from the mean equator and equinox of J2000.0 to the mean ecliptic and equinox of J2000.0. This is simply a rotation around the x-axis (the direction toward the mean equinox of J2000.0) by the angle ϵ_0, the mean obliquity of J2000.0. After the rotation, the fundamental plane is the ecliptic of J2000.0.

2. A rotation around the new z-axis (the direction toward the ecliptic pole of J2000.0) by the angle $-\psi_A$, the amount of precession of the equator from J2000.0 to t.
3. A rotation around the new x-axis (the direction along the intersection of the mean equator of t with the ecliptic of J2000.0) by the angle $-\omega_A$, the obliquity of the mean equator of t with respect to the ecliptic of J2000.0. After the rotation, the fundamental plane is the mean equator of t.
4. A rotation around the new z-axis (the direction toward the mean celestial pole of t) by the angle χ_A, accounting for the precession of the ecliptic along the mean equator of t. After the rotation, the new x-axis is in the direction of the mean equinox of date.

If we let

$$\begin{aligned} S_1 &= \sin(\epsilon_0) & C_1 &= \cos(\epsilon_0) \\ S_2 &= \sin(-\psi_A) & C_2 &= \cos(-\psi_A) \\ S_3 &= \sin(-\omega_A) & C_3 &= \cos(-\omega_A) \\ S_4 &= \sin(\chi_A) & C_4 &= \cos(\chi_A) \end{aligned} \tag{5.9}$$

then the precession matrix can also be written:

$$\mathbf{P}(t) = \begin{pmatrix} C_4C_2 - S_2S_4C_3 & C_4S_2C_1 + S_4C_3C_2C_1 - S_1S_4S_3 & C_4S_2S_1 + S_4C_3C_2S_1 + C_1S_4S_3 \\ -S_4C_2 - S_2C_4C_3 & -S_4S_2C_1 + C_4C_3C_2C_1 - S_1C_4S_3 & -S_4S_2S_1 + C_4C_3C_2S_1 + C_1C_4S_3 \\ S_2S_3 & -S_3C_2C_1 - S_1C_3 & -S_3C_2S_1 + C_3C_1 \end{pmatrix} \tag{5.10}$$

Existing applications that use the 3-angle precession formulation of Newcomb and Lieske can be easily modified for the IAU 2000A precession, by replacing the current polynomials for the angles ζ_A, z_A, and θ_A with the following:

$$\begin{aligned} \zeta_A &= 2.650545 + 2306.083227\,T + 0.2988499\,T^2 + 0.01801828\,T^3 - 0.000005971\,T^4 \\ &\quad - 0.0000003173\,T^5 \\ z_A &= -2.650545 + 2306.077181\,T + 1.0927348\,T^2 + 0.01826837\,T^3 - 0.000028596\,T^4 \\ &\quad - 0.0000002904\,T^5 \\ \theta_A &= 2004.191903\,T - 0.4294934\,T^2 - 0.04182264\,T^3 - 0.000007089\,T^4 \\ &\quad - 0.0000001274\,T^5 \end{aligned} \tag{5.11}$$

The 3-angle precession matrix is $\mathbf{P}(t) = \mathbf{R}_3(-z_A)\,\mathbf{R}_2(\theta_A)\,\mathbf{R}_3(-\zeta_A)$, but any existing correct construction of $\mathbf{P}$ using these three angles can still be used.

The expression for the mean obliquity of the ecliptic (the angle between the mean equator and ecliptic, or, equivalently, between the ecliptic pole and mean celestial pole of date) is:

$$\epsilon = \epsilon_0 - 46.836769\,T - 0.0001831\,T^2 + 0.00200340\,T^3 - 0.000000576\,T^4 - 0.0000000434\,T^5 \tag{5.12}$$

where, as stated above, $\epsilon_0 = 84381.406$ arcseconds. This expression arises from the precession formulation but is actually used only for nutation. (Almost all of the obliquity rate — the term linear in T — is due to the precession of the ecliptic.)

Where high accuracy is not required, the precession between two dates, t_1 and t_2, not too far apart (i.e., where $|t_2 - t_1| \ll 1$ century), can be approximated using the rates of change of right ascension and declination with respect to the mean equator and equinox of date. These rates are respectively

$$\begin{aligned} m &\approx 4612.16 + 2.78\,T \\ n &\approx 2004.19 - 0.86\,T \end{aligned} \tag{5.13}$$

where the values are in arcseconds per century and T is the number of centuries between J2000.0 and the midpoint of t_1 and t_2. If the dates are expressed as Julian dates, $T = ((t_1 + t_2)/2 - 2451545.0)/36525$. Then, denoting the celestial coordinates at t_1 by (α_1, δ_1) and those at t_2 by (α_2, δ_2),

$$\begin{aligned} \alpha_2 &\approx \alpha_1 + \tau\,(m + n\,\sin\alpha_1\tan\delta_1) \\ \delta_2 &\approx \delta_1 + \tau\,(n\,\cos\alpha_1) \end{aligned} \tag{5.14}$$

where $\tau = t_2 - t_1$, expressed in centuries. These formulas deteriorate in accuracy at high (or low) declinations and should not be used at all for coordinates close to the celestial poles (how close depends on the accuracy requirement and the value of τ) .

5.4.2 Formulas for Nutation

Nutation is conventionally expressed as two small angles, $\Delta\psi$, the nutation in longitude, and $\Delta\epsilon$, the nutation in obliquity. These angles are measured in the ecliptic system of date, which is developed as part of the precession formulation. The angle $\Delta\psi$ is the small change in the position of the equinox along the ecliptic due to nutation, so the effect of nutation on the ecliptic coordinates of a fixed point in the sky is simply to add $\Delta\psi$ to its ecliptic longitude. The angle $\Delta\epsilon$ is the small change in the obliquity of the ecliptic due to nutation. The *true obliquity* of date is $\epsilon' = \epsilon + \Delta\epsilon$. Nutation in obliquity reflects the orientation of the equator in space and does not affect the ecliptic coordinates of a fixed point on the sky.

The angles $\Delta\psi$ and $\Delta\epsilon$ can also be thought of as small shifts in the position of the celestial pole (CIP) with respect to the ecliptic and mean equinox of date. In that coordinate system, and assuming positive values for $\Delta\psi$ and $\Delta\epsilon$, the nutation in longitude shifts the celestial pole westward on the sky by the angle $\Delta\psi\,\sin\epsilon$, decreasing the pole's mean ecliptic longitude by $\Delta\psi$. Nutation in obliquity moves the celestial pole further from the ecliptic pole, i.e., southward in ecliptic coordinates, by $\Delta\epsilon$. (Negative values of $\Delta\psi$ and $\Delta\epsilon$ move the pole eastward and northward in ecliptic coordinates.)

The effect of nutation on the equatorial coordinates (α,δ) of a fixed point in the sky is more complex and is best dealt with through the action of the nutation matrix, $\mathbf{N}(t)$, on the equatorial position vector, $\mathbf{r}_{\mathrm{mean}(t)}$. Where high accuracy is not required, formulas that directly give the changes to α and δ as a function of $\Delta\psi$ and $\Delta\epsilon$ are available and are given at the end of this subsection.

The values of $\Delta\psi$ and $\Delta\epsilon$ are obtained by evaluating rather lengthy trigonometric series, of the general form

$$\Delta\psi = \sum_{i=1}^{N}\left((S_i + \dot{S}_i T)\sin\Phi_i + C'_i\cos\Phi_i\right)$$

$$\Delta\epsilon = \sum_{i=1}^{N} \Big((C_i + \dot{C}_i T) \cos \Phi_i + S'_i \sin \Phi_i \Big) \qquad (5.15)$$

$$\text{where, in each term,} \quad \Phi_i = \sum_{j=1}^{K} M_{i,j}\, \phi_j(T) \qquad (5.16)$$

For the IAU 2000A model, N=1365 and K=14. The 14 $\phi_j(T)$ are the fundamental arguments, which are, except for one, orbital angles. The main time dependence of the nutation series enters through these arguments. The expressions given below are all taken from Simon et al. (1994) and all coefficients are in arcseconds.

The first eight fundamental arguments are the mean heliocentric ecliptic longitudes of the planets Mercury through Neptune:

$$\begin{aligned}
\phi_1 &= 908103.259872 + 538101628.688982\,T \\
\phi_2 &= 655127.283060 + 210664136.433548\,T \\
\phi_3 &= 361679.244588 + 129597742.283429\,T \\
\phi_4 &= 1279558.798488 + 68905077.493988\,T \qquad (5.17) \\
\phi_5 &= 123665.467464 + 10925660.377991\,T \\
\phi_6 &= 180278.799480 + 4399609.855732\,T \\
\phi_7 &= 1130598.018396 + 1542481.193933\,T \\
\phi_8 &= 1095655.195728 + 786550.320744\,T
\end{aligned}$$

In all of these expressions, T is the number of Julian centuries of TDB since 2000 Jan 1, 12^{h} TDB (or, with negligible error, the number of Julian centuries of TT since J2000.0). In some implementations it may be necessary to reduce the resulting angles, which are expressed in arcseconds, to radians in the range 0–2π. The ninth argument is an approximation to the general precession in longitude:

$$\phi_9 = 5028.8200\,T + 1.112022\,T^2 \qquad (5.18)$$

The last five arguments are the same fundamental luni-solar arguments used in previous nutation theories, but with updated expresssions. They are, respectively, l, the mean anomaly of the Moon; l', the mean anomaly of the Sun; F, the mean argument of latitude of the Moon; D, the mean elongation of the Moon from the Sun, and Ω, the mean longitude of the Moon's mean ascending node:

$$\begin{aligned}
\phi_{10} &= l = 485868.249036 + 1717915923.2178\,T + 31.8792\,T^2 + 0.051635\,T^3 - 0.00024470\,T^4 \\
\phi_{11} &= l' = 1287104.79305 + 129596581.0481\,T - 0.5532\,T^2 + 0.000136\,T^3 - 0.00001149\,T^4 \\
\phi_{12} &= F = 335779.526232 + 1739527262.8478\,T - 12.7512\,T^2 - 0.001037\,T^3 + 0.00000417\,T^4 \\
\phi_{13} &= D = 1072260.70369 + 1602961601.2090\,T - 6.3706\,T^2 + 0.006593\,T^3 - 0.00003169\,T^4 \\
\phi_{14} &= \Omega = 450160.398036 - 6962890.5431\,T + 7.4722\,T^2 + 0.007702\,T^3 - 0.00005939\,T^4 \qquad (5.19)
\end{aligned}$$

The first step in evaluating the series for nutation for a given date is to compute the values of all 14 fundamental arguments for the date of interest. This is done only once. Then the nutation terms are evaluated one by one. For each term i, first compute Φ_i according to eq. 5.16, using the 14 integer multipliers, $M_{i,j}$, listed for the term; i.e., sum over $M_{i,j} \times \phi_j$ (where j=1–14). Then the cosine and sine components for the term can be evaluated, as per eq. 5.15, using the listed values of the coefficients S_i, $\dot{S}_i$, C'_i, C_i, $\dot{C}_i$, and S'_i for the term. Generally it is good practice to sum the terms from smallest to largest to preserve precision in the sums.

The entire IAU 2000A nutation series is listed at the end of this circular. About the first half of the series consists of lunisolar terms, which depend only on l, l', F, D, and Ω ($= \phi_{10}$ to ϕ_{14}). In all of these terms, the first nine multipliers are all zero. The generally smaller planetary terms comprise the remainder of the series. As an example of how the individual terms are computed according to eqs. 5.15 and 5.16, term 6 would be evaluated

$$\begin{aligned}
\Delta\psi_6 &= (-0.0516821 + 0.0001226\,T)\sin\Phi_6 - 0.0000524\cos\Phi_6 \\
\Delta\epsilon_6 &= (\ \ 0.0224386 - 0.0000667\,T)\cos\Phi_6 - 0.0000174\sin\Phi_6 \\
&\text{where}\quad \Phi_6 = \phi_{11} + 2\phi_{12} - 2\phi_{13} + 2\phi_{14}
\end{aligned}$$

since $M_{6,1}$ through $M_{6,10}$ are zero, and only ϕ_{11} through ϕ_{14} are therefore relevant for this term. It is assumed that all the ϕ_j have been pre-computed (for all terms) using the appropriate value of T for the date and time of interest. A printed version of a 1365-term nutation series is obviously not the most convenient form for computation; it is given here only for the record, since the full series has not previously appeared in print. As noted earlier, the series is available as a pair of plain-text computer files at URL 14, and the SOFA and NOVAS software packages (URL 7, URL 8) include subroutines for evaluating it. There are also shorter series available where the highest precision is not required. The IERS web site provides, in addition to the full IAU 2000A series, an IAU 2000B series, which has only 77 terms and duplicates the IAU 2000A results to within a milliarcsecond for input times between 1995 and 2050. NOVAS also provides a subroutine that evaluates a truncated series, with 488 terms, that duplicates the full series to 0.1 milliarcsecond accuracy between 1700 and 2300.

Once the nutation series has been evaluated and the values of $\Delta\psi$ and $\Delta\epsilon$ are available, the nutation matrix can be constructed. The nutation matrix is simply $\mathbf{N}(t) = \mathbf{R}_1(-\epsilon')\,\mathbf{R}_3(-\Delta\psi)\,\mathbf{R}_1(\epsilon)$, where, again, $\mathbf{R}_1$ and $\mathbf{R}_3$ are standard rotations about the x and z axes, respectively (see "Abbreviations and Symbols Frequently Used" for precise definitions), and $\epsilon' = \epsilon + \Delta\epsilon$ is the true obliquity (compute ϵ using eq. 5.12). This formulation is comprised of

1. A rotation from the mean equator and equinox of t to the mean ecliptic and equinox of t. This is simply a rotation around the x-axis (the direction toward the mean equinox of t) by the angle ϵ, the mean obliquity of t. After the rotation, the fundamental plane is the ecliptic of t.

2. A rotation around the new z-axis (the direction toward the ecliptic pole of t) by the angle $-\Delta\psi$, the amount of nutation in longitude at t. After the rotation, the new x-axis is in the direction of the true equinox of t.

3. A rotation around the new x-axis (the direction toward the true equinox of t) by the angle $-\epsilon'$, the true obliquity of t. After the rotation, the fundamental plane is the true equator of t, orthogonal to the computed position of the CIP at t.

If we let

$$\begin{array}{rclcrcl} S_1 & = & \sin(\epsilon) & \qquad & C_1 & = & \cos(\epsilon) \\ S_2 & = & \sin(-\Delta\psi) & & C_2 & = & \cos(-\Delta\psi) \\ S_3 & = & \sin(-\epsilon-\Delta\epsilon) & & C_3 & = & \cos(-\epsilon-\Delta\epsilon) \end{array} \tag{5.20}$$

then the nutation matrix can also be written:

$$\mathbf{N}(t) = \begin{pmatrix} C_2 & S_2C_1 & S_2S_1 \\ -S_2C_3 & C_3C_2C_1 - S_1S_3 & C_3C_2S_1 + C_1S_3 \\ S_2S_3 & -S_3C_2C_1 - S_1C_3 & -S_3C_2S_1 + C_3C_1 \end{pmatrix} \tag{5.21}$$

Where high accuracy is not required, coordinates corrected for nutation in right ascension and declination can be obtained from

$$\begin{aligned} \alpha_{\mathrm{t}} &\approx \alpha_{\mathrm{m}} + \Delta\psi\,(\cos\epsilon' + \sin\epsilon'\sin\alpha_{\mathrm{m}}\tan\delta_{\mathrm{m}}) - \Delta\epsilon\,\cos\alpha_{\mathrm{m}}\tan\delta_{\mathrm{m}} \\ \delta_{\mathrm{t}} &\approx \delta_{\mathrm{m}} + \Delta\psi\,\sin\epsilon'\cos\alpha_{\mathrm{m}} + \Delta\epsilon\,\sin\alpha_{\mathrm{m}} \end{aligned} \tag{5.22}$$

where $(\alpha_{\mathrm{m}}, \delta_{\mathrm{m}})$ are coordinates with respect to the mean equator and equinox of date (precession only), $(\alpha_{\mathrm{t}}, \delta_{\mathrm{t}})$ are the corresponding coordinates with respect to the true equator and equinox of date (precession + nutation), and ϵ' is the true obliquity. Note the $\tan\delta_{\mathrm{m}}$ factor in right ascension that makes these formulas unsuitable for use close to the celestial poles.

The traditional formula for the equation of the equinoxes (the difference between apparent and mean sidereal time) is $\Delta\psi\cos\epsilon$, but in recent years this has been superceded by the more accurate version given in eq. 2.14.

5.4.3 Alternative Combined Transformation

The following matrix, $\mathbf{C}(t)$, combines precession, nutation, and frame bias and is used to transform vectors from the GCRS to the Celestial Intermediate Reference System (CIRS). The CIRS is defined by the equator of the CIP and an origin of right ascension called the Celestial Intermediate Origin (CIO). The CIO is discussed extensively in Chapter 6. There, the CIRS is symbolized E_σ; it is analogous to the true equator and equinox of date, but with a different right ascension origin.

The matrix $\mathbf{C}(t)$ is used in the sense

$$\mathbf{r}_{\mathrm{CIRS}} = \mathbf{C}(t)\,\mathbf{r}_{\mathrm{GCRS}} \tag{5.23}$$

and the components of $\mathrm{C}(t)$, as given in the IERS Conventions (2003) and *The Astronomical Almanac*, are

$$\mathbf{C}(t) = \mathbf{R}_3(-s)\begin{pmatrix} 1-bX^2 & -bXY & -X \\ -bXY & 1-bY^2 & -Y \\ X & Y & 1-b(X^2+Y^2) \end{pmatrix} \tag{5.24}$$

where X and Y are the dimensionless coordinates of the CIP in the GCRS (unit vector components), $b = 1/(1+Z)$, $Z = \sqrt{1-X^2-Y^2}$, and s is the *CIO locator*, a small angle described in Chapter 6. All of these quantities are functions of time. $\mathbf{R}_3$ is a standard rotation around the z axis; see "Abbreviations and Symbols Frequently Used" for a precise definition.

5.4.4 Observational Corrections to Precession-Nutation

The IERS still publishes daily values of the observed celestial pole offsets, despite the vast improvement to the pole position predictions given by the IAU 2000A precession-nutation model. The offsets now have magnitudes generally less than 1 mas. The fact that they are non-zero is due in part to an effect of unpredictable amplitude and phase called the free core nutation (or nearly diurnal free wobble), caused by the rotation of the fluid core of the Earth inside the ellipsoidal cavity that it occupies. The free core nutation appears as a very small nutation component with a period of about 430 days. Any other effects not accounted for in the adopted precession-nutation model will also appear in the celestial pole offsets. In any event, the celestial pole offsets are now so small that many users may now decide to ignore them. However, it is worth noting again that, by definition, the Celestial Intermediate Pole (CIP) includes these observed offsets.

The IERS now publishes celestial pole offsets with respect to the IAU 2000A precession-nutation model only as dX and dY — corrections to the pole's computed unit vector components X and Y in the GCRS (see eq. 5.6 and following notes). The IERS pole offsets are published in units of milliarcseconds but they can be converted to dimensionless quantities by dividing them by the number of milliarcseconds in one radian, 206264806.247... . Then, the observationally corrected values of X and Y are

$$X_{\text{cor}} = X + dX \qquad \text{and} \qquad Y_{\text{cor}} = Y + dY \tag{5.25}$$

The corrected values, expressed as dimensionless quantities (unit vector components), are used, e.g., in the matrix **C** given in sections 5.4.3 and 6.5.3. That is, in eqs. 5.24 and 6.18, assume $X = X_{\text{cor}}$ and $Y = Y_{\text{cor}}$.

The ecliptic-based pole celestial offsets, $d\psi$ and $d\epsilon$, which are used to correct the nutation theory's output angles $\Delta\psi$ and $\Delta\epsilon$, are no longer supplied (actually, they are supplied but only for the old pre-2000 precession-nutation model). Software that has not been coded to use X and Y directly — which includes all software developed prior to 2003 — will need a front-end to convert the IERS dX and dY values to $d\psi$ and $d\epsilon$. A derivation of a conversion algorithm and several options for its implementation (depending on the accuracy desired) are given by Kaplan (2003). Succinctly, given dimensionless dX and dY values for a given date t, let

$$\begin{pmatrix} dX' \\ dY' \\ dZ' \end{pmatrix} = \mathbf{P}(t) \begin{pmatrix} dX \\ dY \\ dZ \end{pmatrix} \tag{5.26}$$

where $\mathbf{P}(t)$ is the precession matrix from J2000.0 to date t, and we can set $dZ = 0$ in this approximation, which holds for only a few centuries around J2000.0. Then we compute the ecliptic-based correction angles in radians using

$$d\psi = dX' / \sin\epsilon \qquad \text{and} \qquad d\epsilon = dY' \tag{5.27}$$

where ϵ is the mean obliquity of the ecliptic of date t, computed according to eq 5.12. The observationally corrected values of $\Delta\psi$ and $\Delta\epsilon$ are obtained simply by adding $d\psi$ and $d\epsilon$, respectively:

$$\Delta\psi_{\text{cor}} = \Delta\psi + d\psi \qquad \text{and} \qquad \Delta\epsilon_{\text{cor}} = \Delta\epsilon + d\epsilon \tag{5.28}$$

where care must be taken to ensure that all angles are expressed in the same units. The corrected values are used in forming the nutation matrix $\mathbf{N}(t)$ and in other nutation-related expressions. That is, in eqs. 5.20 and 5.22, assume $\Delta\psi = \Delta\psi_{\text{cor}}$ and $\Delta\epsilon = \Delta\epsilon_{\text{cor}}$. At the same time, the corrected value of $\Delta\psi$ should be used in forming the equation of the equinoxes using eq. 2.14.

Chapter 6

Modeling the Earth's Rotation

Relevant IAU resolutions: B1.6, B1.7, B1.8 of 2000

Summary Res. 1.8 of 2000 establishes two new reference points in the plane of the moving (instantaneous) equator for the measurement of Earth rotation: the point on the geocentric celestial sphere is called the Celestial Intermediate Origin (CIO) and the point on the surface of the Earth is called the Terrestrial Intermediate Origin (TIO). The CIO and TIO are specific examples of a concept called a *non-rotating origin* that was first described by Guinot (1979, 1981).

The Earth Rotation Angle, θ, is the geocentric angle between the directions of the CIO and TIO, and provides a new way to represent the rotation of the Earth in the transformation from terrestrial to celestial systems or vice versa. Traditionally, Greenwich sidereal time, which is the hour angle of the equinox with respect to the Greenwich meridian, has served this purpose. The CIO and TIO are defined in such a way that θ is a linear function of Universal Time (UT1) and independent of the Earth's precession and nutation; it is a direct measure of the rotational angle of the Earth around the Celestial Intermediate Pole (see Chapter 5). Since none of these statements holds for sidereal time, the scheme based on the CIO, TIO, and θ represents a simplification of the way the rotation of the Earth is treated. In particular, the transformation between Earth-fixed and space-fixed reference systems can now be specified by three rotation matrices that are independent of each other: one for polar motion, one for "pure" rotation (i.e., θ), and one for precession-nutation.

The recent IAU resolutions do not eliminate sidereal time or the use of the equinox as a fundamental reference point. Instead, the resolutions establish an alternative way of dealing with Earth rotation. The comparison between the two schemes can be illuminating. For example, the CIO helps to clarify the relationship between sidereal time and the Earth's rotation, since θ is now the "fast term" in the formula for sidereal time as a function of UT1. The remaining terms comprise the *equation of the origins* and represent the accumulated amount of precession and nutation along the equator as a function of time. The equation of the origins is the length of the arc between the equinox and the CIO.

6.1 A Messy Business

In the computation of the positions of celestial objects with respect to an Earth-fixed system — or, equivalently, in the transformation between terrestrial and celestial coordinate systems — sidereal time has conventionally represented the Earth's rotation about its axis. For example, the hour angle of an object is simply the local apparent sidereal time minus the object's apparent right ascension with respect to the true equator and equinox of date (see section 2.6.2). Once its hour angle and declination are available, the object's zenith distance and azimuth, or its coordinates with respect to some ground-based instrumental system, can be easily obtained. The same result can be accomplished by a direct transformation between the celestial and terrestrial coordinate systems, conventionally represented by a series of rotation matrices, one each for precession, nutation, sidereal time, and polar motion.

Yet there is something untidy about these procedures. The computation of apparent sidereal time mixes quantities related to Earth rotation, precession, and nutation (see eqs. 2.10–2.14). Because sidereal time is defined as the hour angle of the equinox, the precession of the equinox in right ascension must be a part of the expression for sidereal time (the terms in parentheses in eq. 2.12), and the mean sidereal day is thereby shorter than the rotation period of the Earth by about $0^{\mathrm{s}}\!.008$. Nutation also appears, in the equation of the equinoxes (eqs. 2.13 & 2.14). The result is that in the computation of hour angle, precession and nutation enter twice: once in the sidereal time formula and again in the computation of the star's apparent right ascension; the two contributions cancel for stars on the equator. Similarly, in the transformation between the celestial and terrestrial coordinate systems, precession and nutation each enter into two of the rotation matrices, and none of the matrices represents Earth rotation alone.

A consequence of this way of doing things is that whenever improvements are made to the theory of precession, the numerical coefficients in the expression for sidereal time must also change. This was not an issue for most of the twentieth century, since no adjustments were made to the standard precession algorithm, and the expression for mean sidereal time derived from Newcomb's developments was used without much thought given to the matter. It was the change to this expression, necessitated by the adjustment of the precession constant in the IAU (1976) System of Astronomical Constants, that first motivated the search for a fundamental change of procedure. At about the same time, new high-precision observing techniques, such as VLBI and lunar laser ranging, were being applied to the study of all components of the Earth's rotation, and a review of the basic algorithms seemed appropriate. In particular, there was interest in constructing a new geometrical picture and set of expressions for the orientation of the Earth as a function of time that would cleanly separate the effects of rotation, precession and nutation, and polar motion. Furthermore, since VLBI is not sensitive to the equinox, a procedure that used a different reference point seemed desirable.

To bring the Earth's rotation period explicitly into the terrestrial–celestial transformation, we must define an angle of rotation about the Earth's axis. As described in Chapter 5, what we specifically mean by "the Earth's axis" is the line through the geocenter in the direction of the Celestial Intermediate Pole (CIP). The angle of rotation about this axis must be measured with respect to some agreed-upon direction in space. Since the CIP moves small amounts during one rotation of the Earth (~0.1 arcsecond with respect to the stars and ~0.005 arcsecond with respect to the Earth's crust), the reference direction cannot be simply a fixed vector or plane in inertial space. What we need is an appropriate azimuthal[1] origin — a point in the moving equatorial plane, which is orthogonal to the CIP.

6.2 Non-Rotating Origins

The reference point that we define must be such that the rate of change of the Earth's rotation angle, measured with respect to this point, is the angular velocity of the Earth about the CIP. As the CIP moves, the point must move to remain in the equatorial plane; but the point's motion must be such that the measured rotation angle is not contaminated by some component of the motion of the CIP itself.

The concept of a "non-rotating origin" (NRO) on the equator can be applied to any rotating body. The NRO idea was first described by Bernard Guinot (Guinot 1979, 1981) and further developed by Nicole Capitaine and collaborators (Capitaine et al. 1986; Capitaine 1990; Capitaine & Chollet 1991; Capitaine et al. 2000; Capitaine 2000). The condition on the motion of such a point is simple: as the equator moves, the point's instantaneous motion must always be orthogonal to the equator. That is, the point's motion at some time t must be directly toward or away from the position of the pole of rotation at t. Any other motion of the point would have a component *around* the axis/pole and would thus introduce a spurious rate into the measurement of the rotation angle of the body as a function of time. The point is not unique; any arbitrary point on the moving equator could be made to move in the prescribed manner. For the Earth, the difference between the motion of a non-rotating origin and that of the equinox on the geocentric celestial sphere is illustrated in Fig. 6.1.

As illustrated in the figure, the motion of the non-rotating origin, σ, is always orthogonal to the equator, whereas the equinox has a motion along the equator (the precession in right ascension). How do we specify the location of a non-rotating origin? There are three possibilities, outlined in the Formulas section of this chapter. In the most straightforward scheme, one simply uses the GCRS right ascension of σ obtained from a numerical integration (the GCRS is the "geocentric ICRS"). Alternatively, the position of σ can be defined by a quantity, s, that is the difference between the lengths of two arcs on the celestial sphere. Finally, one can specify the location of σ with respect to the equinox, Υ: the equatorial arc $\overline{\Upsilon\sigma}$ is called the *equation of the origins.* Whatever geometry is used, the position of σ ultimately depends on an integral over time, because the defining property of σ is its *motion* — not a static geometrical relationship with other points or planes. The integral involved is fairly simple and depends only on the coordinates of the pole and their derivatives with respect to time. The initial point for the integration can be any point on the moving equator at any time t_0.

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[1] The word "azimuthal" is used in its general sense, referring to an angle measured about the z-axis of a coordinate system.

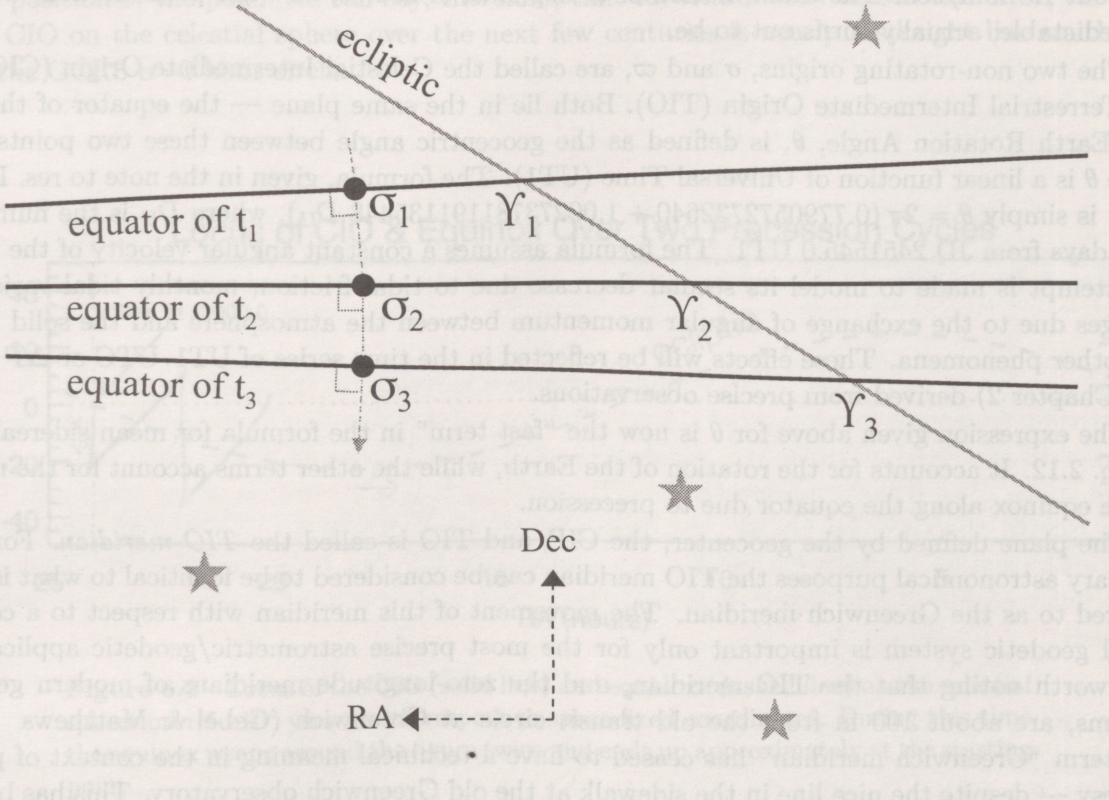

Figure 6.1 Motion of a non-rotating origin, σ, compared with that of the true equinox, Υ. "Snapshots" of the positions of the points are shown at three successive times, t_1, t_2, and t_3. The positions are shown with respect to a geocentric reference system that has no systematic rotation with respect to a set of extragalactic objects. The ecliptic is shown in the figure as fixed, although it, too, has a small motion in inertial space.

So far we have discussed a non-rotating origin only on the celestial sphere, required because of the movement of the CIP in a space-fixed reference system. But there is a corresponding situation on the surface of the Earth. The CIP has motions in both the celestial and terrestrial reference systems. Its motion in the celestial system is precession-nutation and its motion in the terrestrial system is polar motion, or wobble. From the point of view of a conventional geodetic coordinate system "attached" to the surface of the Earth (i.e., defined by the specified coordinates of a group of stations), the CIP wanders around near the geodetic pole in a quasi-circular motion with an amplitude of about 10 meters (0.3 arcsec) and two primary periods, 12 and 14 months. Thus the equator of the CIP has a slight quasi-annual wobble around the geodetic equator. Actually, it is better thought of in the opposite sense: the geodetic equator has a slight wobble with respect to the equator of the CIP. That point of view makes it is a little clearer why a simple "stake in the ground" at the geodetic equator would not be suitable for measuring the Earth rotation angle around the CIP. The situation is orders of magnitude less troublesome than that on the celestial sphere, but for completeness (and very precise applications) it is appropriate to define a terrestrial non-rotating origin, designated ϖ. It stays on the CIP equator, and assuming that the current amplitude of polar motion remains approximately constant, ϖ will bob north and south by about 10 m in geodetic latitude every year or so and will have a secular eastward motion in longitude of about 1.5 mm/cen. The exact motion of ϖ depends, of course, on what polar motion, which is unpredictable, actually turns out to be.

The two non-rotating origins, σ and ϖ, are called the Celestial Intermediate Origin (CIO) and the Terrestrial Intermediate Origin (TIO). Both lie in the same plane — the equator of the CIP. The Earth Rotation Angle, θ, is defined as the geocentric angle between these two points. The angle θ is a linear function of Universal Time (UT1). The formula, given in the note to res. B1.8 of 2000, is simply $\theta = 2\pi\,(0.7790572732640 + 1.00273781191135448\ D_U)$, where D_U is the number of UT1 days from JD 2451545.0 UT1. The formula assumes a constant angular velocity of the Earth: no attempt is made to model its secular decrease due to tidal friction, monthly tidal variations, changes due to the exchange of angular momentum between the atmosphere and the solid Earth, and other phenomena. These effects will be reflected in the time series of UT1–UTC or ΔT values (see Chapter 2) derived from precise observations.

The expression given above for θ is now the "fast term" in the formula for mean sidereal time; see eq. 2.12. It accounts for the rotation of the Earth, while the other terms account for the motion of the equinox along the equator due to precession.

The plane defined by the geocenter, the CIP, and TIO is called the *TIO meridian.* For most ordinary astronomical purposes the TIO meridian can be considered to be identical to what is often referred to as the Greenwich meridian. The movement of this meridian with respect to a conventional geodetic system is important only for the most precise astrometric/geodetic applications. It is worth noting that the TIO meridian, and the zero-longitude meridians of modern geodetic systems, are about 100 m from the old transit circle at Greenwich (Gebel & Matthews 1971). The term "Greenwich meridian" has ceased to have a technical meaning in the context of precise geodesy — despite the nice line in the sidewalk at the old Greenwich observatory. This has become obvious to tourists carrying GPS receivers!

6.3 The Path of the CIO on the Sky

If we take the epoch J2000.0 as the starting epoch for evaluating the integral that provides the position of the CIO, the only mathematical requirement for the initial point is that it lie on the instantaneous (true) equator of that date — its position along the equator is arbitrary. By convention, however, the initial position of the CIO on the instantaneous equator of J2000.0 is set so that equinox-based and CIO-based computations of Earth rotation yield the same answers; we want the hour angle of a given celestial object to be the same, as a function of UT1 (or UTC), no matter how the calculation is done. For this to happen, the position of the CIO of J2000.0 must be at GCRS right ascension $0^\circ\,0'\,00.''002012$. This is about 12.8 arcseconds west of the true equinox of that date.

Since the CIO rides on the instantaneous equator, its primary motion over the next few millenia is southward at the rate of precession in declination, initially 2004 arcseconds per century. Its rate of southward motion is modulated (but never reversed) by the nutation periodicities. Its motion in GCRS right ascension is orders of magnitude less rapid; remember that the CIO has no motion along the instantaneous equator, and the instantaneous equator of J2000.0 is nearly co-planar with the GCRS equator (xy-plane). The motion of the CIO in GCRS right ascension over the next few millenia is dominated by a term proportional to t^3; the GCRS right ascension of the CIO at the beginning of year 2100 is only $0.''068$; at the beginning of 2200 it is $0.''573$; and at the beginning of 2300 it is $1.''941$. Nutation does produce a very slight wobble in the CIO's right ascension, but the influence of the nutation terms is suppressed by several orders of magnitude relative to their effect on the position of the pole. We can say, therefore, that to within a few arcseconds error, the path of the CIO on the celestial sphere over the next few centuries is nearly a straight line southward along the GCRS α=0 hour circle.

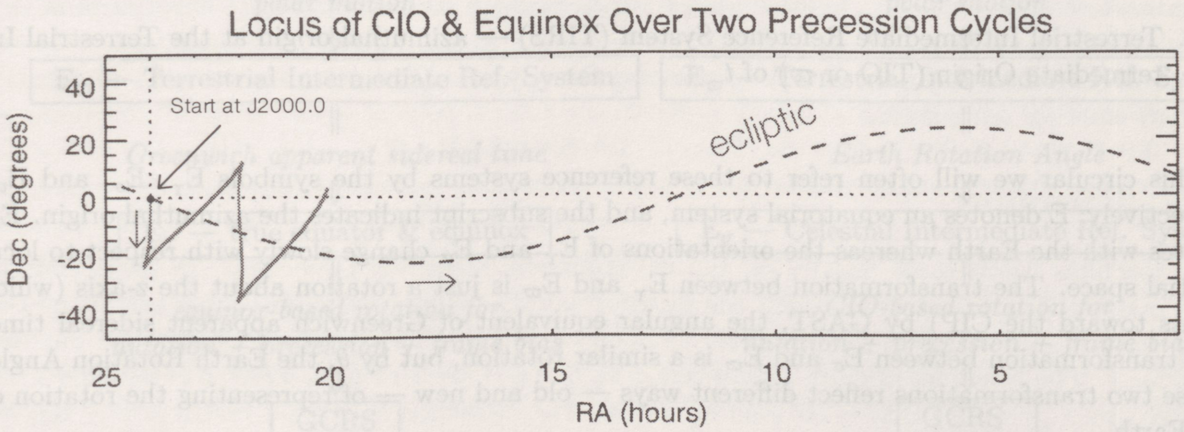

Figure 6.2 Locus of the CIO (solid line) and equinox (dashed line) on the celestial sphere over 5×10^4 years, with respect to space-fixed coordinates. During this time the equinox wraps around the figure twice and ends up approximately at the starting point.

The solid line on the left side of Figure 6.2 indicates the locus of the CIO in the GCRS over 50,000 years — about two precession cycles. The ecliptic is shown as a dashed line. The initial nearly straight southward motion from the starting point at J2000.0 is clearly shown. There are occasional "cusps" in the CIO's motion, where its secular motion comes to a temporary halt before reversing. The first of these stationary points occurs in just over a quarter of a precession cycle, as the section of the moving equator that is farthest south in ecliptic coordinates precesses to near the GCRS α=0 hour circle. At that time, the CIO will exhibit only nutational oscillations around a point that remains fixed on the celestial sphere to within 10 mas for almost a decade. Then its motion resumes, this time northward and westward.[2] The motion of the equinox over the same 50,000-year time period begins at nearly the same point as the CIO (on the plot scale used, the points overlap), but smoothly follows the ecliptic westward (to the right on the plot), wrapping around twice and ending up essentially at the starting point.

6.4 Transforming Vectors Between Reference Systems

The reference points described above allow us to define three geocentric reference systems that share, as a common reference plane, the instantaneous, or true, equator of date. The instantaneous equator is now defined as the plane through the geocenter orthogonal to the direction of the CIP at a given time, t. The three reference systems are:

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[2] There is nothing profound about the stationary points or the dates on which they occur. If we had started the CIO at GCRS right ascension 6^h or 18^h at J2000.0, it would have *started* at a stationary point.

1. True equator and equinox of t — azimuthal origin at the true equinox (Υ) of t
2. Celestial Intermediate Reference System (CIRS) — azimuthal origin at the Celestial Intermediate Origin (CIO or σ) of t
3. Terrestrial Intermediate Reference System (TIRS) — azimuthal origin at the Terrestrial Intermediate Origin (TIO or ϖ) of t

In this circular we will often refer to these reference systems by the symbols E_Υ, E_σ, and E_ϖ, respectively; E denotes an equatorial system, and the subscript indicates the azimuthal origin. E_ϖ rotates with the Earth whereas the orientations of E_Υ and E_σ change slowly with respect to local inertial space. The transformation between E_Υ and E_ϖ is just a rotation about the z-axis (which points toward the CIP) by GAST, the angular equivalent of Greenwich apparent sidereal time. The transformation between E_σ and E_ϖ is a similar rotation, but by θ, the Earth Rotation Angle. These two transformations reflect different ways — old and new — of representing the rotation of the Earth.

A short digression into terrestrial, i.e., geodetic, reference systems is in order here. These systems all have their origin at the geocenter and rotate with the crust of the Earth. Terrestrial latitude, longitude, and height are now most commonly given with respect to a *reference ellipsoid*, an oblate spheroid that approximates the Earth's overall shape (actually, that best fits the *geoid*, a gravitational equipotential surface). The current standard reference elliposid for most purposes is that of the World Geodetic System 1984 (WGS 84), which forms the basis for the coordinates obtained from GPS. The WGS 84 ellipsoid has an equatorial radius of 6,378,137 meters and a polar flattening of 1/298.257223563. For the precise measurement of Earth rotation, however, the International Terrestrial Reference System (ITRS) is used, which was defined by the International Union of Geodesy and Geophysics (IUGG) in 1991. The ITRS is realized for practical purposes by the adopted coordinates and velocities[3] of a large group of observing stations. These coordinates are expressed as geocentric rectangular 3-vectors and thus are not dependent on a reference ellipsoid. The list of stations and their coordinates is referred to as the International Terrestrial Reference Frame (ITRF). The fundamental terrestrial coordinate system is therefore defined in exactly the same way as the fundamental celestial coordinate system (see Chapter 3): a prescription is given for an idealized coordinate system (the ITRS or the ICRS), which is realized in practice by the adopted coordinates of a group of reference points (the ITRF stations or the ICRF quasars). The coordinates may be refined as time goes on but the overall system is preserved. It is important to know, however, that the ITRS/ITRF is consistent with WGS 84 to within a few centimeters; thus for all astronomical purposes the GPS-obtained coordinates of instruments can be used with the algorithms presented here.

Our goal is to be able to transform an arbitrary vector (representing for example, an instrumental position, axis, boresight, or baseline) from the ITRS ($\approx$WGS 84$\approx$GPS) to the GCRS. The three equatorial reference systems described above — E_Υ, E_σ, and E_ϖ — are waypoints, or intermediate stops, in that process. The complete transformations are:

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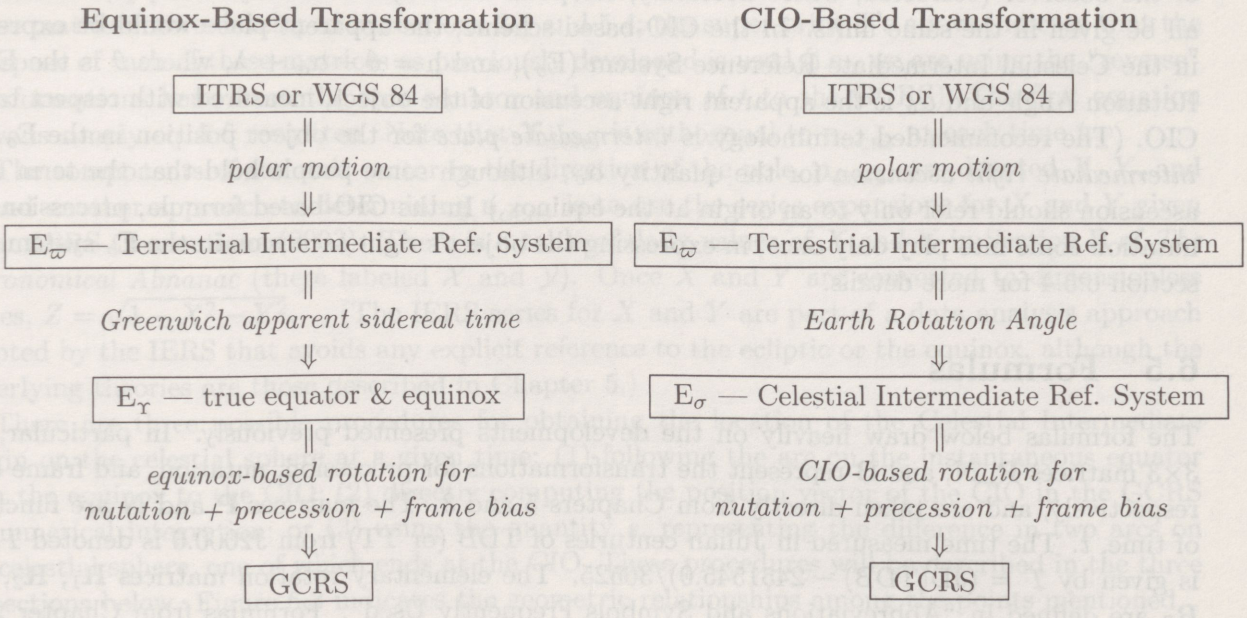
[3] The velocities are quite small and are due to plate tectonics and post-glacial rebound.


Equinox-Based Transformation
CIO-Based Transformation
ITRS or WGS 84
ITRS or WGS 84
polar motion
polar motion
E_ϖ — Terrestrial Intermediate Ref. System
E_ϖ — Terrestrial Intermediate Ref. System
Greenwich apparent sidereal time
Earth Rotation Angle
E_Υ — true equator & equinox
E_σ — Celestial Intermediate Ref. System
equinox-based rotation for nutation + precession + frame bias
CIO-based rotation for nutation + precession + frame bias
GCRS
GCRS

which are equivalent. That is, given the same input vector, the same output vector will result from the two procedures. In the CIO-based transformation, the three sub-transformations (for polar motion, Earth Rotation Angle, and nutation/precession/frame bias) are independent. That is not true for the equinox-based method, because apparent sidereal time incorporates precession and nutation. Each of the two methods could be made into a single matrix, and the two matrices must be numerically identical. That means that the use of the CIO in the second method does not increase the precision of the result but simply allows for a mathematical redescription of the overall transformation — basically, a re-sorting of the effects to be taken into account. This redescription of the transformation provides a clean separation of the three main aspects of Earth rotation, and recognizes that the observations defining modern reference systems are not sensitive to the equinox. It thus yields a more straightforward conceptual view and facilitates a simpler observational analysis for Earth-rotation measurements and Earth-based astrometry.

These transformations are all rotations that pivot around a common point, the geocenter. Although developed for observations referred to the geocenter, the same set of rotations can be applied to observations made from a specific point on the surface of the Earth. (This follows from the assumption that all points in or on the Earth are rigidly attached to all other points. The actual non-rigidity — e.g., Earth tides — is handled as a separate correction.) In such a case, the computations for parallax, light-bending, aberration, etc., must take into account the non-geocentric position and velocity of the observer. Then the final computed coordinates are referred, not to the GCRS, but rather to a proper reference system (in the terminology of relativity) of the observer.

We return to the more familiar problem mentioned at the beginning of the chapter: the computation of local hour angle. In the usual equinox-based scheme, the apparent place[4] of the star or planet is expressed with respect to the true equator and equinox of date (E_Υ). The local hour angle is just $h = \text{GAST} - \alpha_\Upsilon + \lambda$, where GAST is Greenwich apparent sidereal time, α_Υ is the apparent right ascension of the object, measured with respect to the true equinox, and λ is the longitude of the observer (corrected, where necessary, for polar motion). Obviously these quantities must all be given in the same units. In the CIO-based scheme, the apparent place would be expressed in the Celestial Intermediate Reference System (E_σ), and $h = \theta - \alpha_\sigma + \lambda$, where θ is the Earth Rotation Angle and α_σ is the apparent right ascension of the object, measured with respect to the CIO. (The recommended terminology is *intermediate place* for the object position in the E_σ and *intermediate right ascension* for the quantity α_σ, although some people hold that the term right ascension should refer only to an origin at the equinox.) In the CIO-based formula, precession and nutation come into play only once, in expressing the object's right ascension in the E_σ system. See section 6.5.4 for more details.

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[4] See footnote 1 on page 41.

6.5 Formulas

The formulas below draw heavily on the developments presented previously. In particular, the 3×3 matrices **P**, **N**, and **B** represent the transformations for precession, nutation, and frame bias, respectively, and are taken directly from Chapters 5 and 3. The matrices **P** and **N** are functions of time, t. The time measured in Julian centuries of TDB (or TT) from J2000.0 is denoted T and is given by $T = (\mathrm{JD(TDB)} - 2451545.0)/36525$. The elementary rotation matrices $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ are defined in "Abbreviations and Symbols Frequently Used". Formulas from Chapter 2 for sidereal time and the Earth Rotation Angle are used. Explanations of, and formulas for the time scales UT1, TT, and TDB are also found in Chapter 2.

The ultimate objective is to express "local" Earth-fixed vectors, representing geographic positions, baselines, instrumental axes and boresights, etc., in the GCRS, where they can be related to the proper coordinates[5] of celestial objects. As mentioned above, the "GCRS" is a reference system that can be thought of as the "geocentric ICRS". Celestial coordinates in the GCRS are obtained from basic ICRS data (which are barycentric) by applying the usual algorithms for proper place; see section 1.3. In this chapter we will be working entirely in a geocentric system and the GCRS will be obtained from a series of rotations that start with an ordinary Earth-fixed geodetic system.

6.5.1 Location of Cardinal Points

We will start by establishing the positions of three cardinal points within the GCRS: the Celestial Intermediate Pole (CIP), the true equinox of date (Υ), and the Celestial Intermediate Origin (CIO). The unit vectors toward these points will be designated $\mathbf{n}_{\mathrm{GCRS}}$, $\boldsymbol{\Upsilon}_{\mathrm{GCRS}}$, and $\boldsymbol{\sigma}_{\mathrm{GCRS}}$, respectively. As the Earth precesses and nutates in local inertial space, these points are in continual motion.

The CIP and the equinox can easily be located in the GCRS at any time t simply by recognizing that they are, respectively, the z- and x-axes of the true equator and equinox of date system (E_Υ) at t. The unit vectors therefore are:

$$\begin{aligned} \text{CIP} \quad : \quad \mathbf{n}_{\mathrm{GCRS}}(t) &= \mathbf{B}^{\mathrm{T}}\,\mathbf{P}^{\mathrm{T}}(t)\,\mathbf{N}^{\mathrm{T}}(t)\begin{pmatrix}0\\0\\1\end{pmatrix} \\ \text{Equinox:} \quad \boldsymbol{\Upsilon}_{\mathrm{GCRS}}(t) &= \mathbf{B}^{\mathrm{T}}\,\mathbf{P}^{\mathrm{T}}(t)\,\mathbf{N}^{\mathrm{T}}(t)\begin{pmatrix}1\\0\\0\end{pmatrix} \end{aligned} \tag{6.1}$$

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[5] See footnote 1 on page 41.

where the matrix **B** accounts for the GCRS frame bias (same as for the ICRS) and the matrices $\mathbf{P}(t)$ and $\mathbf{N}(t)$ provide the transformations for precession and nutation, respectively, at time t. These matrices were developed in sections 3.5 and 5.4; the superscript T's above indicate that the transpose of each of these matrices as previously developed is used (i.e., we are using the "reverse" transformations here, from the true equator and equinox of t to the GCRS). The first equation above is simply eq. 5.6 rewritten. Note that $\boldsymbol{\Upsilon}_{\text{GCRS}}$ is orthogonal to $\mathbf{n}_{\text{GCRS}}$ at each time t.

The components of the unit vector in the direction of the pole, $\mathbf{n}_{\text{GCRS}}$, are denoted X, Y, and Z, and another approach to determining $\mathbf{n}_{\text{GCRS}}$ is to use the series expansions for X and Y given in the IERS Conventions (2003). There is a table of daily values of X and Y in Section B of *The Astronomical Almanac* (there labeled $\mathcal{X}$ and $\mathcal{Y}$). Once X and Y are converted to dimensionless values, $Z = \sqrt{1 - X^2 - Y^2}$. (The IERS series for X and Y are part of a data analysis approach adopted by the IERS that avoids any explicit reference to the ecliptic or the equinox, although the underlying theories are those described in Chapter 5.)

There are three possible procedures for obtaining the location of the Celestial Intermediate Origin on the celestial sphere at a given time: (1) following the arc on the instantaneous equator from the equinox to the CIO; (2) directly computing the position vector of the CIO in the GCRS by numerical integration; or (3) using the quantity s, representing the difference in two arcs on the celestial sphere, one of which ends at the CIO. These procedures will be described in the three subsections below. Figure 6.3 indicates the geometric relationships among the points mentioned.

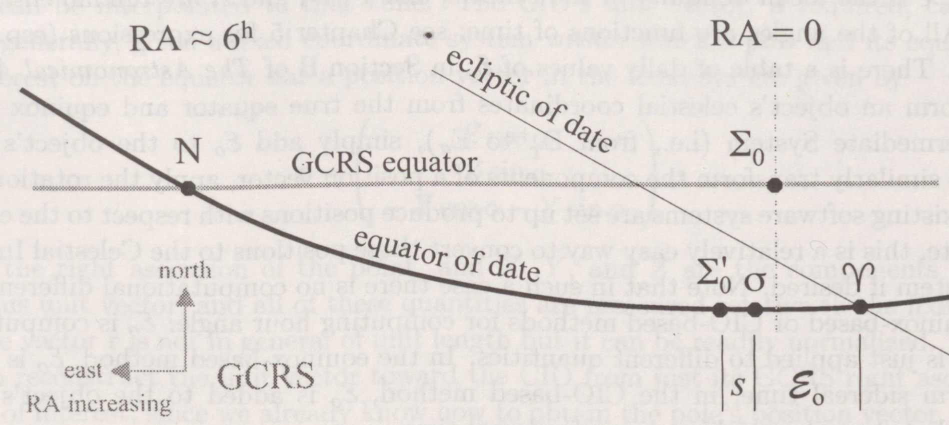

Figure 6.3 Relationship between various points involved in locating the CIO. The figure approximates the relative positions at 2020.0, although the spacings are not to scale. The point labeled σ is the CIO; Υ is the true equinox; Σ_0 is the GCRS right ascension origin; N is the ascending node of the instantaneous (true) equator of date on the GCRS equator; and Σ'_0 is the point on the instantaneous equator that is the same distance from N as Σ_0. As shown, the quantities s and $\mathcal{E}_o$ are respectively positive and negative. The motion of the instantaneous equator (which is orthogonal to the CIP) is generally southward (down in the figure) near RA=0, which tends to move the equinox westward (right) and the CIO very slightly eastward (left) with respect to the GCRS.

6.5.1.1 CIO Location Relative to the Equinox

The arc on the instantaneous (true) equator of date t from the CIO to the equinox is called the *equation of the origins* and is the right ascension of the true equinox relative to the CIO (or, minus the true right ascension of the CIO). The equation of the origins is also the difference $\theta - \text{GAST}$. It therefore equals the accumulated precession-nutation of the equinox in right ascension, given by the sum of the terms in parentheses from eq. 2.12 and the equation of the equinoxes given in eq. 2.14 (all times –1). The equation of the origins in arcseconds therefore is:

$$\begin{aligned}
\mathcal{E}_o = \; & - \; 0.014506 - 4612.156534\,T - 1.3915817\,T^2 + 0.00000044\,T^3 \\
& + \; 0.000029956\,T^4 + 0.0000000368\,T^5 \quad - \Delta\psi \cos\epsilon \\
& - \; 0.00264096 \sin(\Omega) \\
& - \; 0.00006352 \sin(2\Omega) \\
& - \; 0.00001175 \sin(2F - 2D + 3\Omega) \\
& - \; 0.00001121 \sin(2F - 2D + \Omega) \\
& + \; 0.00000455 \sin(2F - 2D + 2\Omega) \\
& - \; 0.00000202 \sin(2F + 3\Omega) \\
& - \; 0.00000198 \sin(2F + \Omega) \\
& + \; 0.00000172 \sin(3\Omega) \\
& + \; 0.00000087\,T \sin(\Omega) + \cdots
\end{aligned} \tag{6.2}$$

where T is the number of centuries of TDB (or TT) from J2000.0; $\Delta\psi$ is the nutation in longitude, in arcseconds; ϵ is the mean obliquity of the ecliptic; and F, D, and Ω are fundamental luni-solar arguments. All of the angles are functions of time; see Chapter 5 for expressions (esp. eqs. 5.12, 5.15, & 5.19). There is a table of daily values of $\mathcal{E}_o$ in Section B of *The Astronomical Almanac.*

To transform an object's celestial coordinates from the true equator and equinox of t to the Celestial Intermediate System (i.e., from E_Υ to E_σ), simply add $\mathcal{E}_o$ to the object's true right ascension. To similarly transform the components of a position vector, apply the rotation $\mathbf{R}_3(-\mathcal{E}_o)$. Since many existing software systems are set up to produce positions with respect to the equator and equinox of date, this is a relatively easy way to convert those positions to the Celestial Intermediate Reference System if desired. Note that in such a case there is no computational difference in using either the equinox-based or CIO-based methods for computing hour angle: $\mathcal{E}_o$ is computed in both methods and is just applied to different quantities. In the equinox-based method, $\mathcal{E}_o$ is subtracted from θ to form sidereal time; in the CIO-based method, $\mathcal{E}_o$ is added to the object's true right ascension so that θ can be used in place of sidereal time.

The position of the CIO in the GCRS, $\boldsymbol{\sigma}_{\text{GCRS}}$, can be established by taking the position vector of the equinox in the GCRS, $\boldsymbol{\Upsilon}_{\text{GCRS}}$, and rotating it counterclockwise by the angle $-\mathcal{E}_o$ (i.e., clockwise by $\mathcal{E}_o$) about the axis $\mathbf{n}_{\text{GCRS}}$. Equivalently, establish the orthonormal basis triad of the equator-and-equinox system within the GCRS: $\boldsymbol{\Upsilon}_{\text{GCRS}}$, $(\mathbf{n}_{\text{GCRS}} \times \boldsymbol{\Upsilon}_{\text{GCRS}})$, and $\mathbf{n}_{\text{GCRS}}$. Then

$$\boldsymbol{\sigma}_{\text{GCRS}} = \boldsymbol{\Upsilon}_{\text{GCRS}} \cos\mathcal{E}_o - (\mathbf{n}_{\text{GCRS}} \times \boldsymbol{\Upsilon}_{\text{GCRS}}) \sin\mathcal{E}_o \tag{6.3}$$

6.5.1.2 CIO Location from Numerical Integration

As described above, a non-rotating origin can be described as a point on the moving equator whose instantaneous motion is always orthogonal to the equator. A simple geometric construction based on this definition yields the following differential equation for the motion of a non-rotating origin:

$$\dot{\boldsymbol{\sigma}}(t) = -\Big(\boldsymbol{\sigma}(t) \cdot \dot{\mathbf{n}}(t)\Big)\, \mathbf{n}(t) \tag{6.4}$$

That is, if we have a model for the motion of the pole, $\mathbf{n}(t)$, the path of the non-rotating origin is described by $\boldsymbol{\sigma}(t)$, once an initial point on the equator, $\boldsymbol{\sigma}(t_0)$, is chosen. Conceptually and practically, it is simple to integrate this equation, using, for example, a standard 4th-order Runge-Kutta integrator. For the motions of the real Earth, fixed step sizes of order 0.5 day work quite well, and the integration is quite robust. This is actually a one-dimensional problem carried out in three dimensions, since we know the non-rotating origin remains on the equator; we really need only to know where along the equator it is. Therefore, two constraints can be applied at each step: $|\boldsymbol{\sigma}| = 1$ and $\boldsymbol{\sigma} \cdot \mathbf{n} = 0$. See Kaplan (2005).

The above equation is quite general, and to get the specific motion of the CIO, each of the vectors in the above equation is expressed with respect to the GCRS, i.e., $\boldsymbol{\sigma}(t) \rightarrow \boldsymbol{\sigma}_{\text{GCRS}}(t)$, $\mathbf{n}(t) \rightarrow \mathbf{n}_{\text{GCRS}}(t)$, and $\dot{\mathbf{n}}(t) \rightarrow \dot{\mathbf{n}}_{\text{GCRS}}(t)$. The pole's position, $\mathbf{n}_{\text{GCRS}}(t)$, is given by the first expression in eq. 6.1. The pole's motion, $\dot{\mathbf{n}}_{\text{GCRS}}(t)$, can be obtained by numerical differentiation of the pole's position. By numerically integrating the above equation, we obtain a time series of unit vectors, $\boldsymbol{\sigma}_{\text{GCRS}}(t_i)$, where each i is an integration step. The fact that this is actually just a one-dimensional problem means that it is sufficient to store as output the CIO right ascensions (with respect to the GCRS), using eq. 5.2 to decompose the $\boldsymbol{\sigma}_{\text{GCRS}}(t_i)$ vectors. In this way, the integration results in a tabulation of CIO right ascensions at discrete times. For example, see URL 17, where the times are expressed as TDB Julian dates and the right ascensions are in arcseconds. This file runs from years 1700 to 2300 at 1.2-day intervals.

When we need to obtain the position of the CIO for some specific time, the file of CIO right ascensions can be interpolated to that time. The CIO's unit vector, if required, can be readily computed: generally, given a fixed coordinate system within which a pole and its equator move, a point of interest on the equator has a position vector in the fixed system given by

$$\mathbf{r} = \begin{pmatrix} Z\cos\alpha \\ Z\sin\alpha \\ -X\cos\alpha - Y\sin\alpha \end{pmatrix} \tag{6.5}$$

where α is the right ascension of the point, and X, Y, and Z are the components of the pole's instantaneous unit vector, and all of these quantities are measured relative to the fixed coordinate system. The vector $\mathbf{r}$ is not in general of unit length but it can be readily normalized. This formula allows us to reconstruct the unit vector toward the CIO from just its GCRS right ascension value at the time of interest, since we already know how to obtain the pole's position vector in the GCRS for that time.

Eq. 6.4 can be also made to yield the locus of the Terrestrial Intermediate Origin (TIO), simply by referring all the vectors to the ITRS — a rotating geodetic system — rather than the GCRS. In this case, therefore, $\boldsymbol{\sigma}(t) \rightarrow \boldsymbol{\varpi}_{\text{ITRS}}(t)$, $\mathbf{n}(t) \rightarrow \mathbf{n}_{\text{ITRS}}(t)$, and $\dot{\mathbf{n}}(t) \rightarrow \dot{\mathbf{n}}_{\text{ITRS}}(t)$. The path of the CIP within the ITRS ($\mathbf{n}_{\text{ITRS}}(t)$) is what we call polar motion (usually specified by the parameters x_p and y_p), and is fundamentally unpredictable. The integration can therefore only be accurately done for past times, using observed pole positions. A computed future path of the TIO on the surface of the Earth depends on the assumption that the two major periodicities observed in polar motion will continue at approximately the current amplitude.

6.5.1.3 CIO Location from the Arc-Difference *s*

On the celestial sphere, the Earth's instantaneous (moving) equator intersects the GCRS equator at two nodes. Let N be the ascending node of the instantaneous equator on the GCRS equator. We can define a scalar quantity $s(t)$ that represents the *difference* between the length of the arc from

N westward to the CIO (on the instantaneous equator) and the length of the arc from N westward to the GCRS origin of right ascension (on the GCRS equator). The quantity s is called the *CIO locator*. If σ represents the CIO and Σ_0 represents the right ascension origin of the GCRS (the direction of the GCRS x-axis), then

$$s = \overline{\sigma\mathrm{N}} - \overline{\Sigma_0\mathrm{N}} \tag{6.6}$$

See Fig. 6.3, where the points Σ_0 and Σ_0' are equidistant from the node N. The quantity s is seen to be the "extra" length of the arc on the instantaneous equator from N to σ, the position of the CIO. The value of s is fundamentally obtained from an integral,

$$s(t) = -\int_{t_0}^{t} \frac{X(t)\dot{Y}(t) - Y(t)\dot{X}(t)}{1 + Z(t)}\, dt + s_0 \tag{6.7}$$

where $X(t)$, $Y(t)$, and $Z(t)$ are the three components of the unit vector, $\mathbf{n}_{\mathrm{GCRS}}(t)$, toward the celestial pole (CIP). See, e.g., Capitaine et al. (2000) or the IERS Conventions (2003). The constant of integration, s_0, has been set to ensure that the equinox-based and CIO-based computations of Earth rotation yield the same answers; s_0=94 μas (IERS Conventions 2003). Effectively, the constant adjusts the position of the CIO on the equator and is thus part of the arc $\overline{\sigma\mathrm{N}}$. For practical purposes, the value of s at any given time is provided by a series expansion, given in Table 5.2c of the IERS Conventions (2003). Software to evaluate this series is available at the IERS Conventions web site and is also part of the SOFA package. There is a table of daily values of s in Section B of *The Astronomical Almanac*.

At any time, t, the the unit vector toward the node N is simply $\mathbf{N}_{\mathrm{GCRS}} = (-Y, X, 0)/\sqrt{X^2 + Y^2}$ (where we are no longer explicitly indicating the time dependence of X and Y). To locate the CIO, we rearrange eq. 6.6 to yield the arc length $\overline{\sigma\mathrm{N}}$:

$$\overline{\sigma\mathrm{N}} = s + \overline{\Sigma_0\mathrm{N}} = s + \arctan(X/(-Y)) \tag{6.8}$$

The location of the CIO is then obtained by starting at the node N and moving along the instantaneous equator of t through the arc $\overline{\sigma\mathrm{N}}$. That is, $\boldsymbol{\sigma}_{\mathrm{GCRS}}$ can be constructed by taking the position vector of the node N in the GCRS, $\mathbf{N}_{\mathrm{GCRS}}$, and rotating it counterclockwise by the angle $-\overline{\sigma\mathrm{N}}$ (i.e., clockwise by $\overline{\sigma\mathrm{N}}$) about the axis $\mathbf{n}_{\mathrm{GCRS}}$. Equivalently,

$$\boldsymbol{\sigma}_{\mathrm{GCRS}} = \mathbf{N}_{\mathrm{GCRS}}\ \cos(\overline{\sigma\mathrm{N}}) - (\mathbf{n}_{\mathrm{GCRS}} \times \mathbf{N}_{\mathrm{GCRS}}) \sin(\overline{\sigma\mathrm{N}}) \tag{6.9}$$

The three methods for determining the position of the CIO in the GCRS are numerically the same to within several microarcseconds (μas) over six centuries centered on J2000.0. We now have formulas in hand for obtaining the positions of the three cardinal points on the sky — the CIP, the CIO, and the equinox — that are involved in the ITRS-to-GCRS (terrestrial-to-celestial) transformations. In the following, it is assumed that $\mathbf{n}_{\mathrm{GCRS}}$, $\boldsymbol{\sigma}_{\mathrm{GCRS}}$, and $\boldsymbol{\Upsilon}_{\mathrm{GCRS}}$ are known vectors for some time t of interest.

6.5.2 Geodetic Position Vectors and Polar Motion

Vectors representing the geocentric positions of points on or near the surface of the Earth are of the general form

$$\mathbf{r} = \begin{pmatrix} (aC+h)\cos\phi_G\cos\lambda_G \\ (aC+h)\cos\phi_G\sin\lambda_G \\ (aS+h)\sin\phi_G \end{pmatrix} \tag{6.10}$$

where λ_G is the geodetic longitude, ϕ_G is the geodetic latitude, and h is the height. These coordinates are measured with respect to a reference ellipsoid, fit to the equipotential surface that effectively defines mean sea level. The ellipsoid has an equatorial radius of a and a flattening factor f. The quantities C and S depend on the flattening:

$$C = 1/\sqrt{\cos^2\phi_G + (1-f)^2\sin^2\phi_G} \qquad S = (1-f)^2\,C \tag{6.11}$$

A complete description of geodetic concepts, reference ellipsoids, and computations is beyond the scope of this circular, but a brief summary can be found in Section K of *The Astronomical Almanac* and a more thorough account is given in Chapter 4 of the Explanatory Supplement (1992). More information can be found in any introductory textbook on geodesy. Suffice it here to say that the reference ellipsoid for GPS is WGS 84, with $a = 6378137$ m and $f = 1/298.257223563$. For astronomical purposes it can be assumed that WGS 84 is a good approximation to the International Terrestrial Reference System (ITRS) described previously in this chapter. That is, GPS provides a realization of the ITRS.

It is worth noting that modern space techniques often measure geocentric positions in rectangular coordinates directly, without using a reference ellipsoid. Also, not all vectors of interest represent geographic locations. Vectors representing instrumental axes, baselines, and boresights are often of more interest to astronomers and these can usually be easily expressed in the same geodetic system as the instrument location. All Earth-fixed vectors, regardless of what they represent, are subject to the same transformations described below. The ITRS is the assumed starting point for these transformations, even though in most cases astronomers will be using vectors in some system that approximates the ITRS.

For astronomical applications, we must correct ITRS vectors for polar motion (also called wobble). In current terminology, this is a transformation from the ITRS to the Terrestrial Intermediate Reference System (TIRS; in this circular it is designated E_ϖ), and is the first transformation shown in the flowcharts on page 56. Polar motion is the small quasi-periodic excursion of the geodetic pole from the pole of rotation, or, more precisely stated, the excursion of the ITRS z-axis from the CIP. It is described by the parameters x_p and y_p, which are the coordinates of the CIP in the ITRS, and which generally amount to a few tenths of an arcsecond. Daily values of x and y, the observed pole coordinates, are published by the IERS (see, e.g., IERS Bulletin A). These published values should, for the most precise applications (<1 mas), be augmented by very small predictable components to polar motion, with periods $\leq$1.2 days. These extra components are evaluated and added after interpolation of the published x and y values — see section 5.4.2 of the IERS Conventions (2003). The sum (total polar motion) is x_p and y_p. However, most users will be able to assume $x_p = x$ and $y_p = y$.

The transformation we seek not only must reorient the pole from the ITRS z-axis to the CIP, it also must move the origin of longitude very slightly from the ITRS x-axis to the Terrestrial Intermediate Origin (TIO). The latter shift is so tiny that its magnitude can be given by an approximate formula, linear in time, based on the two main circular components of polar motion as observed over the last few decades. That shift is $s' = -47\,\mu\mathrm{as}\,T$, where T is the time (either TT or TDB) in centuries from J2000.0 (Lambert & Bizouard 2002). Since 47 μas amounts to 1.5 mm on the surface of the Earth, this correction is entirely negligible for most purposes. Nevertheless, we include it here for completeness.

The ITRS to E_ϖ transformation then is accomplished using the polar motion (wobble) matrix $\mathbf{W}(t)$:

$$\mathbf{r}_{\mathrm{E}_\varpi} = \mathbf{W}(t)\,\mathbf{r}_{\mathrm{ITRS}} \tag{6.12}$$

where

$$\mathbf{W}(t) = \mathbf{R}_3(-s')\,\mathbf{R}_2(x_p)\,\mathbf{R}_1(y_p) \tag{6.13}$$

If we let

$$\begin{aligned} S_x &= \sin(x_p) & C_x &= \cos(x_p) \\ S_y &= \sin(y_p) & C_y &= \cos(y_p) \\ S_s &= \sin(-s') & C_s &= \cos(-s') \end{aligned} \tag{6.14}$$

then the wobble matrix can also be written:

$$\mathbf{W}(t) = \begin{pmatrix} C_xC_s & S_xS_yC_s + C_yS_s & -S_xC_yC_s + S_yS_s \\ -C_xS_s & -S_xS_yS_s + C_yC_s & S_xC_yS_s + S_yC_s \\ S_x & -C_xS_y & C_xC_y \end{pmatrix} \approx \begin{pmatrix} 1 & -s' & -x_p \\ s' & 1 & y_p \\ x_p & -y_p & 1 \end{pmatrix} \tag{6.15}$$

where the form on the right is a first-order approximation. Due to the smallness of the angles involved, the first-order matrix is quite adequate for most applications — further, s' can be set to zero.

6.5.3 Complete Terrestrial to Celestial Transformation

The transformations corresponding to the two flowcharts on page 56 are

$$\begin{aligned} \text{Equinox-based transformation:} \quad \mathbf{r}_{\mathrm{GCRS}} &= \mathbf{B}^{\mathrm{T}}\,\mathbf{P}^{\mathrm{T}}\,\mathbf{N}^{\mathrm{T}}\,\mathbf{R}_3(-\mathrm{GAST})\,\mathbf{W}\,\mathbf{r}_{\mathrm{ITRS}} \\ \text{CIO-based transformation:} \quad \mathbf{r}_{\mathrm{GCRS}} &= \mathbf{C}^{\mathrm{T}}\,\mathbf{R}_3(-\theta)\,\mathbf{W}\,\mathbf{r}_{\mathrm{ITRS}} \end{aligned} \tag{6.16}$$

where all of the matrices except $\mathbf{B}$ are time-dependent. GAST is Greenwich apparent sidereal time and θ is the Earth Rotation Angle; formulas are given in section 2.6.2 (eqs. 2.10–2.14). The matrices, working from right to left, perform the following sub-transformations:

$\mathbf{W}$	ITRS to E_ϖ
$\mathbf{R}_3(-\mathrm{GAST})$	E_ϖ to E_Υ
$\mathbf{B}^{\mathrm{T}}\,\mathbf{P}^{\mathrm{T}}\,\mathbf{N}^{\mathrm{T}}$	E_Υ to GCRS
$\mathbf{R}_3(-\theta)$	E_ϖ to E_σ
$\mathbf{C}^{\mathrm{T}}$	E_σ to GCRS

The three "E" reference systems were described on page 55.

The matrix $\mathbf{C}$ has not yet been developed in this chapter. One form of the matrix $\mathbf{C}$ was introduced in section 5.4.3 for the transformation from the GCRS to E_σ. We use the transpose here because we are interested in the opposite transformation. $\mathbf{C}$ or $\mathbf{C}^{\mathrm{T}}$ is easy to construct because we already have the three basis vectors of E_σ expressed in the GCRS: the z-axis is toward $\mathbf{n}_{\mathrm{GCRS}}$, the CIP; the x-axis is toward $\boldsymbol{\sigma}_{\text{GCRS}}$, the CIO; and the y-axis is toward $\mathbf{n}_{\text{GCRS}} \times \boldsymbol{\sigma}_{\text{GCRS}}$. Call the latter vector $\mathbf{y}_{\text{GCRS}}$. Then:

$$\mathbf{C}^{\mathrm{T}} = \begin{pmatrix} \boldsymbol{\sigma}_{\text{GCRS}} & \mathbf{y}_{\text{GCRS}} & \mathbf{n}_{\text{GCRS}} \end{pmatrix} = \begin{pmatrix} \sigma_1 & y_1 & n_1 \\ \sigma_2 & y_2 & n_2 \\ \sigma_3 & y_3 & n_3 \end{pmatrix} = \begin{pmatrix} \sigma_1 & y_1 & X \\ \sigma_2 & y_2 & Y \\ \sigma_3 & y_3 & Z \end{pmatrix} \tag{6.17}$$

where X, Y, and Z are the CIP coordinates, expressed as dimensionless quantities. As in section 5.4.3, the matrix can also be constructed using only X and Y, together with the CIO locator, s:

$$\mathbf{C}^{\mathrm{T}} = \begin{pmatrix} 1-bX^2 & -bXY & X \\ -bXY & 1-bY^2 & Y \\ -X & -Y & 1-b(X^2+Y^2) \end{pmatrix} \mathbf{R}_3(s) \tag{6.18}$$

where $b = 1/(1+Z)$ and $Z = \sqrt{1-X^2-Y^2}$. The latter form is taken from the IERS Conventions (2003), Chapter 5, where $\mathbf{C}^{\mathrm{T}}$ is called $Q(t)$. The two constructions of $\mathbf{C}^{\mathrm{T}}$ are numerically the same.

6.5.4 Hour Angle

The local hour angle of a celestial object is given by

$$\begin{array}{rl} \text{Equinox-based formula:} & \left(\begin{array}{rcl} \mathrm{h} & = & \mathrm{GAST} - \alpha_{\Upsilon} + \lambda \\ \mathrm{h} & = & \theta - \alpha_{\sigma} + \lambda \end{array} \right. \\ \text{CIO-based formula:} & \end{array} \tag{6.19}$$

where GAST is Greenwich apparent sidereal time, θ is the Earth Rotation Angle, and λ is the longitude of the observer. The quantities involved can be expressed in either angle or time units as long as they are consistent. The right ascension in the two cases is expressed with respect to different origins: α_{Υ} is the apparent right ascension of the object, measured with respect to the true equinox, and α_{σ} is the apparent right ascension of the object, measured with respect to the CIO. That is, the coordinates of the object are expressed in system E_{Υ} in the equinox-based formula and in system E_{σ} in the CIO-based formula. Since both systems share the same equator — the instantaneous equator of date, orthogonal to the CIP — the apparent declination of the object is the same in the two cases.

The two formulas in 6.19 are equivalent, which can be seen by substituting, in the equinox-based formula, GAST $= \theta - \mathcal{E}_o$ and $\alpha_{\Upsilon} = \alpha_{\sigma} - \mathcal{E}_o$, where $\mathcal{E}_o$ is the equation of the origins.

The longitude of the observer, λ, is expressed in the E_{ϖ} system, that is, it is corrected for polar motion. Using the first-order form of the matrix $\mathbf{W}$, given in eq. 6.15 (and assuming s'=0), it is straightforward to derive eq. 2.16 for λ. Using notation consistent with that used in this chapter, this equation is

$$\lambda \equiv \lambda_{\mathrm{E}_{\varpi}} = \lambda_{\text{ITRS}} + \Big(x_p \sin\lambda_{\text{ITRS}} + y_p \cos\lambda_{\text{ITRS}} \Big) \tan\phi_{\text{ITRS}} / 3600 \tag{6.20}$$

where λ_{ITRS} and ϕ_{ITRS} are the ITRS (geodetic) longitude and latitude of the observer, with λ_{ITRS} in degrees; and x_p and y_p are the coordinates of the pole (CIP), in arcseconds. This formula is approximate and should not be used for places at polar latitudes.

The corresponding equation for the latitude, ϕ, corrected for polar motion is

$$\phi \equiv \phi_{\mathrm{E}_{\varpi}} = \phi_{\text{ITRS}} + \Big(x_p \cos\lambda_{\text{ITRS}} - y_p \sin\lambda_{\text{ITRS}} \Big) / 3600 \tag{6.21}$$

although this equation is not needed in hour angle computations; it is given here only for completeness.

The common notion of hour angle can be expressed more precisely using concepts introduced in Chapters 5 and 6. The local hour angle of an object is the angle between two planes: the plane containing the geocenter, the CIP, and the observer; and the plane containing the geocenter, the CIP, and the object. Hour angle increases with time and is positive when the object is west of the observer as viewed from the geocenter. The two planes define meridians on the celestial sphere that meet at the CIP. From the point of view of the observer, the CIP is not, in general, exactly at the geodetic north point, which is the direction toward the ITRS z-axis. The azimuths of the two directions differ by as much as $\sqrt{x_p^2 + y_p^2}/\cos\phi_{\text{ITRS}}$, depending on time of day. This difference is small (usually <1 arcsecond) and often negligible for practical applications. The plane defining the astronomical Greenwich meridian (from which Greenwich hour angles are measured) can be understood to contain the geocenter, the CIP, and TIO; there, $\lambda \equiv \lambda_{\text{E}_\varpi} = 0$. This plane is now called the TIO meridian.

The CIO-based formula for hour angle is quite simple to use (since θ is linear with time) if we have the coordinates of the object expressed in system E_σ. Fortunately, this is straightforward if we have the object's coordinates expressed in the GCRS, because we also have the basis vectors of E_σ expressed in the GCRS. If the object's vector in the GCRS is $\mathbf{r}_{\text{GCRS}}$, then the object's vector in E_σ is simply

$$\mathbf{r}_{\text{E}_\sigma} = \mathbf{C}\,\mathbf{r}_{\text{GCRS}} = \begin{pmatrix} \mathbf{r}_{\text{GCRS}} \cdot \boldsymbol{\sigma}_{\text{GCRS}} \\ \mathbf{r}_{\text{GCRS}} \cdot \mathbf{y}_{\text{GCRS}} \\ \mathbf{r}_{\text{GCRS}} \cdot \mathbf{n}_{\text{GCRS}} \end{pmatrix} \quad \text{where} \quad \mathbf{y}_{\text{GCRS}} = \mathbf{n}_{\text{GCRS}} \times \boldsymbol{\sigma}_{\text{GCRS}} \tag{6.22}$$

Then

$$\alpha_\sigma = \arctan\left(\frac{\mathbf{r}_{\text{GCRS}} \cdot \mathbf{y}_{\text{GCRS}}}{\mathbf{r}_{\text{GCRS}} \cdot \boldsymbol{\sigma}_{\text{GCRS}}}\right) \tag{6.23}$$

As a specific case, we know the position vector of the equinox, $\boldsymbol{\Upsilon}_{\text{GCRS}}$. Applying eq. 6.23 to $\boldsymbol{\Upsilon}_{\text{GCRS}}$ and using it in the second formula of 6.19, with λ=0, we obtain the hour angle of the equinox at the Greenwich (or TIO) meridian. But this is the definition of Greenwich Apparent Sidereal Time. Therefore,

$$\text{GAST} = \theta - \arctan\left(\frac{\boldsymbol{\Upsilon}_{\text{GCRS}} \cdot \mathbf{y}_{\text{GCRS}}}{\boldsymbol{\Upsilon}_{\text{GCRS}} \cdot \boldsymbol{\sigma}_{\text{GCRS}}}\right) \tag{6.24}$$

Evidently, then,

$$\mathcal{E}_o = \arctan\left(\frac{\boldsymbol{\Upsilon}_{\text{GCRS}} \cdot \mathbf{y}_{\text{GCRS}}}{\boldsymbol{\Upsilon}_{\text{GCRS}} \cdot \boldsymbol{\sigma}_{\text{GCRS}}}\right) \tag{6.25}$$

which merely restates the definition of $\mathcal{E}_o$ — the equatorial angle from the CIO to the equinox, i.e., the right ascension of the equinox in system E_σ.

Text of IAU Resolutions of 1997

Adopted at the XXIIIrd General Assembly, Kyoto

Resolution B2 On the International Celestial Reference System (ICRS)

The XXIIIrd International Astronomical Union General Assembly

Considering

(a) That Recommendation VII of Resolution A4 of the 21st General Assembly specifies the coordinate system for the new celestial reference frame and, in particular, its continuity with the FK5 system at J2000.0;

(b) That Resolution B5 of the 22nd General Assembly specifies a list of extragalactic sources for consideration as candidates for the realization of the new celestial reference frame;

(c) That the IAU Working Group on Reference Frames has in 1995 finalized the positions of these candidate extragalactic sources in a coordinate frame aligned to that of the FK5 to within the tolerance of the errors in the latter (see note 1);

(d) That the Hipparcos Catalogue was finalized in 1996 and that its coordinate frame is aligned to that of the frame of the extragalactic sources in (c) with one sigma uncertainties of ± 0.6 milliarcseconds (mas) at epoch J1991.25 and ± 0.25 mas per year in rotation rate;

Noting

That all the conditions in the IAU Resolutions have now been met;

Resolves

(a) That, as from 1 January 1998, the IAU celestial reference system shall be the International Celestial Reference System (ICRS) as specified in the 1991 IAU Resolution on reference frames and as defined by the International Earth Rotation Service (IERS) (see note 2);

(b) That the corresponding fundamental reference frame shall be the International Celestial Reference Frame (ICRF) constructed by the IAU Working Group on Reference Frames;

(c) That the Hipparcos Catalogue shall be the primary realization of the ICRS at optical wavelengths;

(d) That IERS should take appropriate measures, in conjunction with the IAU Working Group on reference frames, to maintain the ICRF and its ties to the reference frames at other wavelengths.

Note 1: IERS 1995 Report, Observatoire de Paris, p. II-19 (1996).

Note 2: "The extragalactic reference system of the International Earth Rotation Service (ICRS)", Arias, E.F. et al. A & A 303, 604 (1995).

Resolution B4 On Non-Rigid Earth Nutation Theory

The XXIIIrd International Astronomical Union General Assembly

Recognizing

that the International Astronomical Union and the International Union of Geodesy and Geophysics Working Group (IAU-IUGG WG) on Non-rigid Earth Nutation Theory has met its goal by identifying the remaining geophysical and astronomical phenomena that must be modeled before an accurate theory of nutation for a non-rigid Earth can be adopted, and

that, as instructed by IAU Recommendation C1 in 1994, the International Earth Rotation Service (IERS) has published in the IERS Conventions (1996) an interim precession-nutation model that matches the observations with an uncertainty of ± 1 milliarcsecond (mas),

endorses

the conclusions of the IAU-IUGG WG on Non-rigid Earth Nutation Theory given in the appendix,

requests

the IAU-IUGG WG on Non-rigid Earth Nutation Theory to present a detailed report to the next IUGG General Assembly (August 1999), at which time the WG will be discontinued,

and urges

the scientific community to address the following questions in the future:

- completion of a new rigid Earth nutation series with the additional terms necessary for the theory to be complete to within ± 5 microarcseconds, and

- completion of a new non-rigid Earth transfer function for an Earth initially in non-hydrostatic equilibrium, incorporating mantle inelasticity and a Free Core Nutation period in agreement with the observations, and taking into account better modeling of the fluid parts of the planet, including dissipation.

APPENDIX

The WG on Non-rigid Earth Nutation Theory has quantified the problems in the nutation series adopted by the IAU in 1980 by noting:

(1) that there is a difference in the precession rate of about −3.0 milliarcseconds per year (mas/year) between the value observed by Very Long Baseline Interferometry (VLBI) and Lunar Laser Ranging (LLR) and the adopted value,

(2) that the obliquity has been observed (by VLBI and LLR) to change at a rate of about −0.24 mas/year, although there is no such change implied by the 1980 precession-nutation theory,

(3) that, in addition to these trends, there are observable peak-to-peak differences of up to 20 milliarcseconds (mas) between the nutation observed by VLBI and LLR and the nutation adopted by the IAU in 1980,

(4) that these differences correspond to spectral amplitudes of up to several mas, and

(5) that the differences between observation and theory are well beyond the present observational accuracy.

The WG has recognized the improvements made in the modeling of these quantities, and recommends, in order to derive a more precise nutation model, at the mas level in spectral amplitudes and at a few mas level in the peak to peak analysis, the use of models:

(1) based on a new non-rigid Earth transfer function for an Earth initially in non-hydrostatic equilibrium, incorporating mantle inelasticity, a core-mantle-boundary flattening giving a Free Core Nutation (FCN) period in agreement with the observed value, and a global Earth dynamical flattening in agreement with the observed precession, and

(2) based on a new rigid Earth nutation series which takes into account the following perturbing effects:

1. in lunisolar ephemerides: indirect planetary effects, lunar inequality, J2-tilt, planetary-tilt, secular variations of the amplitudes, effects of precession and nutation,
2. in the perturbing bodies to be considered: in addition to the Moon and the Sun, the direct planetary effects of Venus, Jupiter, Mars, and Saturn, should be included,
3. in the order of the external potential to be considered: J3 and J4 effects for the Moon, and
4. in the theory itself: effects of the tri-axiality of the Earth, relativistic effects and second order effects.

The WG recognizes that this new generation of models still has some imperfections, the principal one being poor modeling of the dissipation in the core and of certain effects of the ocean and the atmosphere, and urges the scientific community to address these questions in the future.

The WG recognizes that, due to the remaining imperfections of the present theoretical nutation models, the nutation series published in the IERS Conventions (1996), following 1994 IAU recommendation C1, still provides the users with the best nutation series. This IERS model being based on observations of the celestial pole offset, the WG supports the recommendation that the scientific community continue VLBI and LLR observations to provide accurate estimations of nutation, precession and rate of change in obliquity.

Text of IAU Resolutions of 2000

Adopted at the XXIVth General Assembly, Manchester

Resolution B1.1 Maintenance and Establishment of Reference Frames and Systems

The XXIVth International Astronomical Union

Noting

1. that Resolution B2 of the XXIIIrd General Assembly (1997) specifies that "the fundamental reference frame shall be the International Celestial Reference Frame (ICRF) constructed by the IAU Working Group on Reference Frames,"

2. that Resolution B2 of the XXIIIrd General Assembly (1997) specifies "That the Hipparcos Catalogue shall be the primary realization of the ICRS at optical wavelengths", and

3. the need for accurate definition of reference systems brought about by unprecedented precision, and

Recognizing

1. the importance of continuing operational observations made with Very Long Baseline Interferometry (VLBI) to maintain the ICRF,

2. the importance of VLBI observations to the operational determination of the parameters needed to specify the time-variable transformation between the International Celestial and Terrestrial Reference Frames,

3. the progressive shift between the Hipparcos frame and the ICRF, and

4. the need to maintain the optical realization as close as possible to the ICRF

Recommends

1. that IAU Division I maintain the Working Group on Celestial Reference Systems formed from Division I members to consult with the International Earth Rotation Service (IERS) regarding the maintenance of the ICRS,

2. that the IAU recognize the International VLBI service (IVS) for Geodesy and Astrometry as an

IAU Service Organization,

3. that an official representative of the IVS be invited to participate in the IAU Working Group on Celestial Reference Systems,

4. that the IAU continue to provide an official representative to the IVS Directing Board,

5. that the astrometric and geodetic VLBI observing programs consider the requirements for maintenance of the ICRF and linking to the Hipparcos optical frame in the selection of sources to be observed (with emphasis on the Southern Hemisphere), design of observing networks, and the distribution of data, and

6. that the scientific community continue with high priority ground- and space-based observations (a) for the maintenance of the optical Hipparcos frame and frames at other wavelengths and (b) for the links of the frames to the ICRF.

Resolution B1.2 Hipparcos Celestial Reference Frame

The XXIVth International Astronomical Union

Noting

1. that Resolution B2 of the XXIIIrd General Assembly (1997) specifies, "That the Hipparcos Catalogue shall be the primary realization of the International Celestial Reference System (ICRS) at optical wavelengths,"

2. the need for this realization to be of the highest precision,

3. that the proper motions of many of the Hipparcos stars known, or suspected, to be multiple are adversely affected by uncorrected orbital motion,

4. the extensive use of the Hipparcos Catalogue as reference for the ICRS in extension to fainter stars,

5. the need to avoid confusion between the International Celestial Reference Frame (ICRF) and the Hipparcos frame, and

6. the progressive shift between the Hipparcos frame and the ICRF,

Recommends

1. that Resolution B2 of the XXIIIrd IAU General Assembly (1997) be amended by excluding from the optical realization of the ICRS all stars flagged C, G, O, V and X in the Hipparcos Catalogue, and

2. that this modified Hipparcos frame be labeled the Hipparcos Celestial Reference Frame (HCRF).

Resolution B1.3 Definition of Barycentric Celestial Reference System and Geocentric Celestial Reference System

The XXIVth International Astronomical Union

Considering

1. that the Resolution A4 of the XXIst General Assembly (1991) has defined a system of space-time coordinates for (a) the solar system (now called the Barycentric Celestial Reference System, (BCRS)) and (b) the Earth (now called the Geocentric Celestial Reference System (GCRS)), within the framework of General Relativity,

2. the desire to write the metric tensors both in the BCRS and in the GCRS in a compact and self-consistent form, and

3. the fact that considerable work in General Relativity has been done using the harmonic gauge that was found to be a useful and simplifying gauge for many kinds of applications,

Recommends

1. the choice of harmonic coordinates both for the barycentric and for the geocentric reference systems.

2. writing the time-time component and the space-space component of the barycentric metric $g_{\mu\nu}$ with barycentric coordinates (t, **x**) (t = Barycentric Coordinate Time (TCB)) with a single scalar potential w(t,**x**) that generalizes the Newtonian potential, and the space-time component with a vector potential w^i(t, **x**); as a boundary condition it is assumed that these two potentials vanish far from the solar system,

explicitly,

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4},$$

$$g_{0i} = -\frac{4}{c^3} w^i,$$

$$g_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2} w\right),$$

with

$$w(t, \mathbf{x}) = G \int d^3x' \frac{\sigma(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \frac{1}{2c^2} G \frac{\partial^2}{\partial t^2} \int d^3x' \sigma(t, \mathbf{x}') |\mathbf{x} - \mathbf{x}'|$$

$$w^i(t, \mathbf{x}) = G \int d^3x' \frac{\sigma^i(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}.$$

Here, σ and σ^i are the gravitational mass and current densities, respectively.

3. writing the geocentric metric tensor $G_{\alpha\beta}$ with geocentric coordinates (T, **X**) (T= Geocentric Coordinate Time (TCG)) in the same form as the barycentric one but with potentials W(T, **X**) and W^a(T, **X**); these geocentric potentials should be split into two parts — potentials W and W^a arising from the gravitational action of the Earth and external parts W_{ext} and W^a_{ext} due to tidal and inertial effects; the external parts of the metric potentials are assumed to vanish at the geocenter

and admit an expansion into positive powers of **X**,

explicitly,

$$G_{00} = -1 + \frac{2W}{c^2} - \frac{2W^2}{c^4},$$

$$G_{0a} = -\frac{4}{c^3} W^a,$$

$$G_{ab} = \delta_{ab}\left(1 + \frac{2}{c^2} W\right).$$

The potentials W and W^a should be split according to

$$W(T, \mathbf{X}) = W_E(T, \mathbf{X}) + W_{ext}(T, \mathbf{X}),$$

$$W^a(T, \mathbf{X}) = W^a_E(T, \mathbf{X}) + W^a_{ext}(T, \mathbf{X}).$$

The Earth's potentials W$_E$ and W^a_E are defined in the same way as w and w^i but with quantities calculated in the GCRS with integrals taken over the whole Earth.

4. using, if accuracy requires, the full post-Newtonian coordinate transformation between the BCRS and the GCRS as induced by the form of the corresponding metric tensors,

explicitly, for the kinematically non-rotating GCRS (T=TCG, t=TCB, $r^i_E \equiv x^i - x^i_E(t)$ and a summation from 1 to 3 over equal indices is implied),

$$T = t - \frac{1}{c^2}[A(t) + v^i_E r^i_E] + \frac{1}{c^4}[B(t) + B^i(t) r^i_E + B^{ij}(t) r^i_E r^j_E + C(t, \mathbf{x})] + O(c^{-5}),$$

$$X^a = \delta_{ai}\left[r^i_E + \frac{1}{c^2}\left(\frac{1}{2} v^i_E v^j_E r^j_E + w_{ext}(\mathbf{x_E}) r^i_E + r^i_E a^j_E r^j_E - \frac{1}{2} a^i_E r^2_E\right)\right] + O(c^{-4}),$$

where

$$\frac{d}{dt} A(t) = \frac{1}{2} v^2_E + w_{ext}(\mathbf{x_E}),$$

$$\frac{d}{dt} B(t) = -\frac{1}{8} v^4_E - \frac{3}{2} v^2_E w_{ext}(\mathbf{x_E}) + 4 v^i_E w^i_{ext}(\mathbf{x_E}) + \frac{1}{2} w^2_{ext}(\mathbf{x_E}),$$

$$B^i(t) = -\frac{1}{2} v^2_E v^i_E + 4 w^i_{ext}(\mathbf{x_E}) - 3 v^i_E w_{ext}(\mathbf{x_E}),$$

$$B^{ij}(t) = -v^i_E \delta_{aj} Q^a + 2 \frac{\partial}{\partial x^j} w^i_{ext}(\mathbf{x_E}) - v^i_E \frac{\partial}{\partial x^j} w_{ext}(\mathbf{x_E}) + \frac{1}{2} \delta^{ij} \dot{w}_{ext}(\mathbf{x_E}),$$

$$C(t, \mathbf{x}) = -\frac{1}{10} r^2_E (\dot{a}^i_E r^i_E).$$

Here x^i_E, v^i_E, and a^i_E are the barycentric position, velocity and acceleration vectors of the Earth, the dot stands for the total derivative with respect to t, and

$$Q^a = \delta_{ai}\left[\frac{\partial}{\partial x_i} w_{ext}(\mathbf{x_E}) - a^i_E\right].$$

The external potentials, w_{ext} and w^i_{ext}, are given by

$$w_{ext} = \sum_{A \neq E} w_A, \quad w^i_{ext} = \sum_{A \neq E} w^i_A,$$

where E stands for the Earth and w_A and w^i_A are determined by the expressions for w and w^i with integrals taken over body A only.

Notes

It is to be understood that these expressions for w and w^i give g_{00} correct up to $O(c^{-5})$, g_{0i} up to $O(c^{-5})$, and g_{ij} up to $O(c^{-4})$. The densities σ and σ^i are determined by the components of the energy momentum tensor of the matter composing the solar system bodies as given in the references. Accuracies for $G_{\alpha\beta}$ in terms of c^{-n} correspond to those of $g_{\mu\nu}$.

The external potentials W_{ext} and W^a_{ext} can be written in the form

$$W_{ext} = W_{tidal} + W_{iner},$$

$$W^a_{ext} = W^a_{tidal} + W^a_{iner}.$$

W_{tidal} generalizes the Newtonian expression for the tidal potential. Post-Newtonian expressions for W_{tidal} and W^a_{tidal} can be found in the references. The potentials W_{iner}, W^a_{iner} are inertial contributions that are linear in X^a. The former is determined mainly by the coupling of the Earth's nonsphericity to the external potential. In the kinematically non-rotating Geocentric Celestial Reference System, W^a_{iner} describes the Coriolis force induced mainly by geodetic precession.

Finally, the local gravitational potentials W_E and W^a_E of the Earth are related to the barycentric gravitational potentials w_E and w^i_E by

$$W_E(T,\mathbf{X}) = w_e(t,\mathbf{x})\left(1 + \frac{2}{c^2}v_E^2\right) - \frac{4}{c^2}v_E^i w_E^i(t,\mathbf{x}) + O(c^{-4}),$$

$$W^a_E(T,\mathbf{X}) = \delta_{ai}(w^i_E(t,\mathbf{x}) - v^i_E w_E(t,\mathbf{x})) + O(c^{-2}).$$

References

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Resolution B1.4 Post-Newtonian Potential Coefficients

The XXIVth International Astronomical Union

Considering

1. that for many applications in the fields of celestial mechanics and astrometry a suitable parametrization of the metric potentials (or multipole moments) outside the massive solar-system bodies in the form of expansions in terms of potential coefficients are extremely useful, and

2. that physically meaningful post-Newtonian potential coefficients can be derived from the literature,

Recommends

1. expansion of the post-Newtonian potential of the Earth in the Geocentric Celestial Reference System (GCRS) outside the Earth in the form

$$W_E(T,\mathbf{X}) = \frac{GM_E}{R}\Big[1 + \textstyle\sum_{l=2}^{\infty}\sum_{m=0}^{+l}\left(\frac{R_E}{R}\right)^l P_{lm}(\cos\theta)(C_{lm}^E(T)\cos m\phi + S_{lm}^E(T)\sin m\phi)\Big],$$

where C_{lm}^E and S_{lm}^E are, to sufficient accuracy, equivalent to the post-Newtonian multipole moments introduced in (Damour *et al.*, *Phys. Rev.* *D*, **43**, 3273, 1991), θ and ϕ are the polar angles corresponding to the spatial coordinates X^a of the GCRS and $R = |X|$, and

2. expression of the vector potential outside the Earth, leading to the well-known Lense-Thirring effect, in terms of the Earth's total angular momentum vector $\mathbf{S_E}$ in the form

$$W_E^a(T,\mathbf{X}) = -\frac{G}{2}\frac{(\mathbf{X}\times\mathbf{S_E})^a}{R^3}.$$

Resolution B1.5 Extended relativistic framework for time transformations and realization of coordinate times in the solar system

The XXIVth International Astronomical Union

Considering

1. that the Resolution A4 of the XXIst General Assembly (1991) has defined systems of space-time coordinates for the solar system (Barycentric Reference System) and for the Earth (Geocentric Reference System), within the framework of General Relativity,

2. that Resolution B1.3 entitled "Definition of Barycentric Celestial Reference System and Geocentric Celestial Reference System" has renamed these systems the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS), respectively, and has specified a general framework for expressing their metric tensor and defining coordinate transformations at the first post-Newtonian level,

3. that, based on the anticipated performance of atomic clocks, future time and frequency measurements will require practical application of this framework in the BCRS, and

4. that theoretical work requiring such expansions has already been performed,

Recommends

that for applications that concern time transformations and realization of coordinate times within the solar system, Resolution B1.3 be applied as follows:

1. the metric tensor be expressed as

$$g_{00} = -\Big(1 - \frac{2}{c^2}(w_0(t,\mathrm{x}) + w_L(t,\mathrm{x})) + \frac{2}{c^4}(w_0^2(t,\mathrm{x}) + \Delta(t,\mathrm{x}))\Big),$$

$$g_{0i} = -\tfrac{4}{c^3} w^i(t, \mathbf{x}),$$

$$g_{ij} = \left(1 + \tfrac{2w_0(t,\mathbf{x})}{c^2}\right)\delta_{ij},$$

where (t $\equiv$ Barycentric Coordinate Time (TCB),$\mathbf{x}$) are the barycentric coordinates, $w_0 = G\sum_A M_A/r_A$ with the summation carried out over all solar system bodies A, $\mathbf{r}_A = \mathbf{x} - \mathbf{x}_A$, $\mathbf{x}_A$ are the coordinates of the center of mass of body A, r$_A$ = $|\mathbf{r_A}|$, and where w_L contains the expansion in terms of multipole moments [see their definition in the Resolution B1.4 entitled "Post-Newtonian Potential Coefficients"] required for each body. The vector potential $w^i(t, \mathbf{x}) = \sum_A w^i_A(t, \mathbf{x})$ and the function $\Delta(t, \mathbf{x}) = \sum_A \Delta_A(t, \mathbf{x})$ are given in note 2.

2. the relation between TCB and Geocentric Coordinate Time (TCG) can be expressed to sufficient accuracy by

$$TCB{-}TCG = c^{-2}\left[\int_{t_0}^{t}\left(\frac{v_E^2}{2} + w_{0ext}(\mathbf{x}_E)\right)dt + v_E^i r_E^i\right] - c^{-4}\left[\int_{t_0}^{t}\left(-\frac{1}{8}v_E^4 - \frac{3}{2}v_E^2 w_{0ext}(\mathbf{x}_E) + 4v_E^i w_{ext}^i(\mathbf{x}_E) + \frac{1}{2}w_{0ext}^2(\mathbf{x}_E)\right)dt - \left(3w_{0ext}(\mathbf{x}_E) + \frac{v_E^2}{2}\right)v_E^i r_E^i\right],$$

where v_E is the barycentric velocity of the Earth and where the index ext refers to summation over all bodies except the Earth.

Notes

1. This formulation will provide an uncertainty not larger than 5×10^{-18} in rate and, for quasi-periodic terms, not larger than 5×10^{-18} in rate amplitude and 0.2 ps in phase amplitude, for locations farther than a few solar radii from the Sun. The same uncertainty also applies to the transformation between TCB and TCG for locations within 50000 km of the Earth. Uncertainties in the values of astronomical quantities may induce larger errors in the formulas.

2. Within the above mentioned uncertainties, it is sufficient to express the vector potential $w^i_A(t, \mathbf{x})$ of body A as

$$w^i_A(t, \mathbf{x}) = G\left[\frac{-(\mathbf{r}_A \times \mathbf{S}_A)^i}{2r_A^3} + \frac{M_A v_A^i}{r_A}\right],$$

where $\mathbf{S}_A$ is the total angular momentum of body A and v^i_A is the barycentric coordinate velocity of body A. As for the function $\Delta_A(t, \mathbf{x})$ it is sufficient to express it as

$$\Delta_A(t, \mathbf{x}) = \frac{GM_A}{r_A}\left[-2v_a^2 + \sum_{B \neq A}\frac{GM_B}{r_{BA}} + \frac{1}{2}\left(\frac{(r_A^k v_A^k)^2}{r_A^2} + r_A^k a_A^k\right)\right] + \frac{2Gv_A^k(\mathbf{r}_A \times \mathbf{S}_A)^k}{r_A^3},$$

where $r_{BA} = |\mathbf{x}_B - \mathbf{x}_A|$ and a^k_A is the barycentric coordinate acceleration of body A. In these formulas, the terms in $\mathbf{S}_A$ are needed only for Jupiter ($S \approx 6.9 \times 10^{38} m^2 s^{-1} kg$) and Saturn ($S \approx 1.4 \times 10^{38} m^2 s^{-1} kg$), in the immediate vicinity of these planets.

3. Because the present recommendation provides an extension of the IAU 1991 recommendations valid at the full first post-Newtonian level, the constants L$_C$ and L$_B$ that were introduced in the IAU 1991 recommendations should be defined as $< TCG/TCB >$ = 1 - L$_C$ and $< TT/TCB >$ =

$1 - L_B$, where TT refers to Terrestrial Time and <> refers to a sufficiently long average taken at the geocenter. The most recent estimate of L_C is (Irwin, A. and Fukushima, T., *Astron. Astroph.*, **348**, 642–652, 1999)

$$L_C = 1.48082686741 \times 10^{-8} \pm 2 \times 10^{-17}.$$

From Resolution B1.9 on "Redefinition of Terrestrial Time TT", one infers $L_B = 1.55051976772 \times 10^{-8} \pm 2 \times 10^{-17}$ by using the relation $1 - L_B = (1 - L_C)(1 - L_G)$. L_G is defined in Resolution B1.9.

Because no unambiguous definition may be provided for L_B and L_C, these constants should not be used in formulating time transformations when it would require knowing their value with an uncertainty of order 1×10^{-16} or less.

4. If TCB−TCG is computed using planetary ephemerides which are expressed in terms of a time argument (noted T_{eph}) which is close to Barycentric Dynamical Time (TDB), rather than in terms of TCB, the first integral in Recommendation 2 above may be computed as

$$\int_{t_0}^{t}\left(\frac{v_E^2}{2} + w_{0ext}(\mathbf{x}_E)\right)dt = \left[\int_{T_{eph_0}}^{T_{eph}}\left(\frac{v_E^2}{2} + w_{0ext}(\mathbf{x}_E)\right)dt\right]/(1 - L_B).$$

Resolution B1.6 IAU 2000 Precession-Nutation Model

The XXIVth International Astronomical Union

Recognizing

1. that the International Astronomical Union and the International Union of Geodesy and Geophysics Working Group (IAU-IUGG WG) on 'Non-rigid Earth Nutation Theory' has met its goals by

a. establishing new high precision rigid Earth nutation series, such as (1) SMART97 of Bretagnon *et al.*, 1998, *Astron. Astroph.*, **329**, 329–338; (2) REN2000 of Souchay *et al.*, 1999, *Astron. Astroph. Supl. Ser.*, **135**, 111–131; (3) RDAN97 of Roosbeek and Dehant 1999, *Celest. Mech.*, **70**, 215–253;

b. completing the comparison of new non-rigid Earth transfer functions for an Earth initially in non-hydrostatic equilibrium, incorporating mantle anelasticity and a Free Core Nutation period in agreement with observations,

c. noting that numerical integration models are not yet ready to incorporate dissipation in the core,

d. noting the effects of other geophysical and astronomical phenomena that must be modelled, such as ocean and atmospheric tides, that need further development;

2. that, as instructed by IAU Recommendation C1 in 1994, the International Earth Rotation Service (IERS) will publish in the IERS Conventions (2000) a precession-nutation model that matches the observations with a weighted rms of 0.2 milliarcsecond (mas);

3. that semi-analytical geophysical theories of forced nutation are available which incorporate some or all of the following — anelasticity and electromagnetic couplings at the core-mantle and inner core-outer core boundaries, annual atmospheric tide, geodesic nutation, and ocean tide effects;

4. that ocean tide corrections are necessary at all nutation frequencies; and

5. that empirical models based on a resonance formula without further corrections do also exist;

Accepts

the conclusions of the IAU-IUGG WG on Non-rigid Earth Nutation Theory published by Dehant *et al.*, 1999, *Celest. Mech.* **72(4)**, 245–310 and the recent comparisons between the various possibilities, and

Recommends

that, beginning on 1 January 2003, the IAU 1976 Precession Model and IAU 1980 Theory of Nutation, be replaced by the precession-nutation model IAU 2000A (MHB2000, based on the transfer functions of Mathews, Herring and Buffett, 2000 — submitted to the *Journal of Geophysical Research*) for those who need a model at the 0.2 mas level, or its shorter version IAU 2000B for those who need a model only at the 1 mas level, together with their associated precession and obliquity rates, and their associated celestial pole offsets at J2000.0, to be published in the IERS Conventions 2000, and

Encourages

1. the continuation of theoretical developments of non-rigid Earth nutation series,

2. the continuation of VLBI observations to increase the accuracy of the nutation series and the nutation model, and to monitor the unpredictable free core nutation, and

3. the development of new expressions for precession consistent with the IAU 2000A model.

Resolution B1.7 Definition of Celestial Intermediate Pole

The XXIVth International Astronomical Union

Noting

the need for accurate definition of reference systems brought about by unprecedented observational precision, and

Recognizing

1. the need to specify an axis with respect to which the Earth's angle of rotation is defined,

2. that the Celestial Ephemeris Pole (CEP) does not take account of diurnal and higher frequency variations in the Earth's orientation,

Recommends

1. that the Celestial Intermediate Pole (CIP) be the pole, the motion of which is specified in the Geocentric Celestial Reference System (GCRS, see Resolution B1.3) by motion of the Tisserand mean axis of the Earth with periods greater than two days,

2. that the direction of the CIP at J2000.0 be offset from the direction of the pole of the GCRS in a manner consistent with the IAU 2000A (see Resolution B1.6) precession-nutation model,

3. that the motion of the CIP in the GCRS be realized by the IAU 2000A model for precession and forced nutation for periods greater than two days plus additional time-dependent corrections provided by the International Earth Rotation Service (IERS) through appropriate astro-geodetic observations,

4. that the motion of the CIP in the International Terrestrial Reference System (ITRS) be provided by the IERS through appropriate astro-geodetic observations and models including high-frequency variations,

5. that for highest precision, corrections to the models for the motion of the CIP in the ITRS may be estimated using procedures specified by the IERS, and

6. that implementation of the CIP be on 1 January 2003.

Notes

1. The forced nutations with periods less than two days are included in the model for the motion of the CIP in the ITRS.

2. The Tisserand mean axis of the Earth corresponds to the mean surface geographic axis, quoted B axis, in Seidelmann, 1982, *Celest. Mech.*, **27**, 79–106.

3. As a consequence of this resolution, the Celestial Ephemeris Pole is no longer necessary.

Resolution B1.8 Definition and use of Celestial and Terrestrial Ephemeris Origin

The XXIVth International Astronomical Union

Recognizing

1. the need for reference system definitions suitable for modern realizations of the conventional reference systems and consistent with observational precision,

2. the need for a rigorous definition of sidereal rotation of the Earth,

3. the desirability of describing the rotation of the Earth independently from its orbital motion, and

Noting

that the use of the "non-rotating origin" (Guinot, 1979) on the moving equator fulfills the above conditions and allows for a definition of UT1 which is insensitive to changes in models for precession and nutation at the microarcsecond level,

Recommends

1. the use of the "non-rotating origin" in the Geocentric Celestial Reference System (GCRS) and that this point be designated as the Celestial Ephemeris Origin (CEO) on the equator of the Celestial Intermediate Pole (CIP),

2. the use of the "non-rotating origin" in the International Terrestrial Reference System (ITRS) and that this point be designated as the Terrestrial Ephemeris Origin (TEO) on the equator of the CIP,

3. that UT1 be linearly proportional to the Earth Rotation Angle defined as the angle measured along the equator of the CIP between the unit vectors directed toward the CEO and the TEO,

4. that the transformation between the ITRS and GCRS be specified by the position of the CIP in the GCRS, the position of the CIP in the ITRS, and the Earth Rotation Angle,

5. that the International Earth Rotation Service (IERS) take steps to implement this by 1 January 2003, and

6. that the IERS will continue to provide users with data and algorithms for the conventional transformations.

Note

1. The position of the CEO can be computed from the IAU 2000A model for precession and nutation of the CIP and from the current values of the offset of the CIP from the pole of the ICRF at J2000.0 using the development provided by Capitaine *et al.* (2000).

2. The position of the TEO is only slightly dependent on polar motion and can be extrapolated as done by Capitaine *et al.* (2000) using the IERS data.

3. The linear relationship between the Earth's rotation angle θ and UT1 should ensure the continuity in phase and rate of UT1 with the value obtained by the conventional relationship between Greenwich Mean Sidereal Time (GMST) and UT1. This is accomplished by the following relationship:

$$\theta(UT1) = 2\pi(0.7790572732640 + 1.00273781191135448 \times (Julian\ UT1\ date - 2451545.0))$$

References

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Capitaine, N., Guinot, B., McCarthy, D.D., 2000, "Definition of the Celestial Ephemeris Origin and of UT1 in the International Celestial Reference Frame", *Astron. Astrophys.*, **355**, 398–405.

Resolution B1.9 Re-definition of Terrestrial Time TT

The XXIVth International Astronomical Union

Considering

1. that IAU Resolution A4 (1991) has defined Terrestrial Time (TT) in its Recommendation 4, and

2. that the intricacy and temporal changes inherent to the definition and realization of the geoid are a source of uncertainty in the definition and realization of TT, which may become, in the near future, the dominant source of uncertainty in realizing TT from atomic clocks,

Recommends

that TT be a time scale differing from TCG by a constant rate: $\mathrm{dTT}/\mathrm{dTCG} = 1 - \mathrm{L}_G$, where $\mathrm{L}_G = 6.969290134 \times 10^{-10}$ is a defining constant,

Note

L_G was defined by the IAU Resolution A4 (1991) in its Recommendation 4 as equal to U_G/c^2 where U_G is the geopotential at the geoid. L_G is now used as a defining constant.

Resolution B2 Coordinated Universal Time

The XXIVth International Astronomical Union

Recognizing

1. that the definition of Coordinated Universal Time (UTC) relies on the astronomical observation of the UT1 time scale in order to introduce leap seconds,

2. that the unpredictable leap seconds affects modern communication and navigation systems,

3. that astronomical observations provide an accurate estimate of the secular deceleration of the Earth's rate of rotation

Recommends

1. that the IAU establish a working group reporting to Division I at the General Assembly in 2003 to consider the redefinition of UTC,

2. that this study discuss whether there is a requirement for leap seconds, the possibility of inserting leap seconds at pre-determined intervals, and the tolerance limits for UT1−UTC, and

3. that this study be undertaken in cooperation with the appropriate groups of the International Union of Radio Science (URSI), the International Telecommunications Union (ITU-R), the International Bureau for Weights and Measures (BIPM), the International Earth Rotation Service (IERS) and relevant navigational agencies.

References

Journal abbreviations:

A&A	*Astronomy and Astrophysics*
A&A Supp	*Astronomy and Astrophysics Supplement Series*
AJ	*The Astronomical Journal*
ApJ Supp	*The Astrophysical Journal Supplement Series*
Celest. Mech.	*Celestial Mechanics*
Celest. Mech. Dyn. Astr.	*Celestial Mechanics and Dynamical Astronomy*
Geophys. J. Royal Astron. Soc.	*Geophysical Journal of the Royal Astronomical Society*
J. Geophys. Res.	*Journal of Geophysical Research*
Phys. Rev. D	*Physical Review D*

Page numbers listed after each reference indicate the location of the citations in this circular.

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[14] Nutation series: `http://maia.usno.navy.mil/conv2003/chapter5/tab5.3a.txt` and `http://maia.usno.navy.mil/conv2003/chapter5/tab5.3b.txt` … 40, 47, 88

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[16] Note on (dX,dY) conversion: `http://aa.usno.navy.mil/kaplan/dXdY_to_dpsideps.pdf` … 84

[17] File of CIO RA: `http://aa.usno.navy.mil/software/novas/novas_f/CIO_RA.txt` … 61

IAU 2000A Nutation Series

The nutation series adopted by the IAU, developed by Mathews et al. (2002) (MHB), is listed below in its entirety. It is also available from the IERS as a pair of plain-text computer files at URL 14, although the arrangement of the columns differs from what is presented here. The IERS also provides a Fortran subroutine for evaluating the nutation series, written by P. Wallace, at URL 15. The NOVAS software package includes this subroutine, and the SOFA package contains the same code in a subroutine of a different name. There are also subroutines available that evaluate only a subset of the series terms for applications that do not require the highest accuracy.

There are 1365 terms in the series. The term numbers are arbitrary and are not involved in the computation. As listed below, the first 678 are lunisolar terms and the remaining 687 are planetary terms. In the lunisolar terms, the only fundamental argument multipliers that are non-zero are $M_{i,10}$ through $M_{i,14}$, corresponding to the arguments l, l', F, D, and Ω, respectively. In the planetary terms, there are no rates of change of the coefficients, i.e., $\dot{S}_i$ and $\dot{C}_i$ are zero.

The formulas for evaluating the series are given in section 5.4.2; see eqs. 5.15–5.16 and the following text.

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-17.2064161	-0.0174666	0.0033386	9.2052331	0.0009086	0.0015377
2	0	0	0	0	0	0	0	0	0	0	0	2	-2	2	-1.3170906	-0.0001675	-0.0013696	0.5730336	-0.0003015	-0.0004587
3	0	0	0	0	0	0	0	0	0	0	0	2	0	2	-0.2276413	-0.0000234	0.0002796	0.0978459	-0.0000485	0.0001374
4	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0.2074554	0.0000207	-0.0000698	-0.0897492	0.0000470	-0.0000291
5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0.1475877	-0.0003633	0.0011817	0.0073871	-0.0000184	-0.0001924
6	0	0	0	0	0	0	0	0	0	0	1	2	-2	2	-0.0516821	0.0001226	-0.0000524	0.0224386	-0.0000677	-0.0000174
7	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0.0711159	0.0000073	-0.0000872	-0.0006750	0.0000000	0.0000358
8	0	0	0	0	0	0	0	0	0	0	0	2	0	1	-0.0387298	-0.0000367	0.0000380	0.0200728	0.0000018	0.0000318
9	0	0	0	0	0	0	0	0	0	1	0	2	0	2	-0.0301461	-0.0000036	0.0000816	0.0129025	-0.0000063	0.0000367
10	0	0	0	0	0	0	0	0	0	0	-1	2	-2	2	0.0215829	-0.0000494	0.0000111	-0.0095929	0.0000299	0.0000132
11	0	0	0	0	0	0	0	0	0	0	0	2	-2	1	0.0128227	0.0000137	0.0000181	-0.0068982	-0.0000009	0.0000039
12	0	0	0	0	0	0	0	0	0	-1	0	2	0	2	0.0123457	0.0000011	0.0000019	-0.0053311	0.0000032	-0.0000004
13	0	0	0	0	0	0	0	0	0	-1	0	0	2	0	0.0156994	0.0000010	-0.0000168	-0.0001235	0.0000000	0.0000082
14	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0.0063110	0.0000063	0.0000027	-0.0033228	0.0000000	-0.0000009
15	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	-0.0057976	-0.0000063	-0.0000189	0.0031429	0.0000000	-0.0000075
16	0	0	0	0	0	0	0	0	0	-1	0	2	2	2	-0.0059641	-0.0000011	0.0000149	0.0025543	-0.0000011	0.0000066
17	0	0	0	0	0	0	0	0	0	1	0	2	0	1	-0.0051613	-0.0000042	0.0000129	0.0026366	0.0000000	0.0000078
18	0	0	0	0	0	0	0	0	0	-2	0	2	0	1	0.0045893	0.0000050	0.0000031	-0.0024236	-0.0000010	0.0000020
19	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0.0063384	0.0000011	-0.0000150	-0.0001220	0.0000000	0.0000029
20	0	0	0	0	0	0	0	0	0	0	0	2	2	2	-0.0038571	-0.0000001	0.0000158	0.0016452	-0.0000011	0.0000068
21	0	0	0	0	0	0	0	0	0	0	-2	2	-2	2	0.0032481	0.0000000	0.0000000	-0.0013870	0.0000000	0.0000000
22	0	0	0	0	0	0	0	0	0	-2	0	0	2	0	-0.0047722	0.0000000	-0.0000018	0.0000477	0.0000000	-0.0000025
23	0	0	0	0	0	0	0	0	0	2	0	2	0	2	-0.0031046	-0.0000001	0.0000131	0.0013238	-0.0000011	0.0000059
24	0	0	0	0	0	0	0	0	0	1	0	2	-2	2	0.0028593	0.0000000	-0.0000001	-0.0012338	0.0000010	-0.0000003
25	0	0	0	0	0	0	0	0	0	-1	0	2	0	1	0.0020441	0.0000021	0.0000010	-0.0010758	0.0000000	-0.0000003
26	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0.0029243	0.0000000	-0.0000074	-0.0000609	0.0000000	0.0000013
27	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0.0025887	0.0000000	-0.0000066	-0.0000550	0.0000000	0.0000011
28	0	0	0	0	0	0	0	0	0	0	1	0	0	1	-0.0014053	-0.0000025	0.0000079	0.0008551	-0.0000002	-0.0000045
29	0	0	0	0	0	0	0	0	0	-1	0	0	2	1	0.0015164	0.0000010	0.0000011	-0.0008001	0.0000000	-0.0000001
30	0	0	0	0	0	0	0	0	0	0	2	2	-2	2	-0.0015794	0.0000072	-0.0000016	0.0006850	-0.0000042	-0.0000005

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
31	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0.0021783	0.0000000	0.0000013	-0.0000167	0.0000000	0.0000013
32	0	0	0	0	0	0	0	0	0	1	0	0	-2	1	-0.0012873	-0.0000010	-0.0000037	0.0006953	0.0000000	-0.0000014
33	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	-0.0012654	0.0000011	0.0000063	0.0006415	0.0000000	0.0000026
34	0	0	0	0	0	0	0	0	0	-1	0	2	2	1	-0.0010204	0.0000000	0.0000025	0.0005222	0.0000000	-0.0000015
35	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0.0016707	-0.0000085	-0.0000010	0.0000168	-0.0000001	0.0000010
36	0	0	0	0	0	0	0	0	0	1	0	2	2	2	-0.0007691	0.0000000	0.0000044	0.0003268	0.0000000	0.0000019
37	0	0	0	0	0	0	0	0	0	-2	0	2	0	0	-0.0011024	0.0000000	-0.0000014	0.0000104	0.0000000	0.0000002
38	0	0	0	0	0	0	0	0	0	0	1	2	0	2	0.0007566	-0.0000021	-0.0000011	-0.0003250	0.0000000	-0.0000005
39	0	0	0	0	0	0	0	0	0	0	0	2	2	1	-0.0006637	-0.0000011	0.0000025	0.0003353	0.0000000	0.0000014
40	0	0	0	0	0	0	0	0	0	0	-1	2	0	2	-0.0007141	0.0000021	0.0000008	0.0003070	0.0000000	0.0000004
41	0	0	0	0	0	0	0	0	0	0	0	0	2	1	-0.0006302	-0.0000011	0.0000002	0.0003272	0.0000000	0.0000004
42	0	0	0	0	0	0	0	0	0	1	0	2	-2	1	0.0005800	0.0000010	0.0000002	-0.0003045	0.0000000	-0.0000001
43	0	0	0	0	0	0	0	0	0	2	0	2	-2	2	0.0006443	0.0000000	-0.0000007	-0.0002768	0.0000000	-0.0000004
44	0	0	0	0	0	0	0	0	0	-2	0	0	2	1	-0.0005774	-0.0000011	-0.0000015	0.0003041	0.0000000	-0.0000005
45	0	0	0	0	0	0	0	0	0	2	0	2	0	1	-0.0005350	0.0000000	0.0000021	0.0002695	0.0000000	0.0000012
46	0	0	0	0	0	0	0	0	0	0	-1	2	-2	1	-0.0004752	-0.0000011	-0.0000003	0.0002719	0.0000000	-0.0000003
47	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	-0.0004940	-0.0000011	-0.0000021	0.0002720	0.0000000	-0.0000009
48	0	0	0	0	0	0	0	0	0	-1	-1	0	2	0	0.0007350	0.0000000	-0.0000008	-0.0000051	0.0000000	0.0000004
49	0	0	0	0	0	0	0	0	0	2	0	0	-2	1	0.0004065	0.0000000	0.0000006	-0.0002206	0.0000000	0.0000001
50	0	0	0	0	0	0	0	0	0	1	0	0	2	0	0.0006579	0.0000000	-0.0000024	-0.0000199	0.0000000	0.0000002
51	0	0	0	0	0	0	0	0	0	0	1	2	-2	1	0.0003579	0.0000000	0.0000005	-0.0001900	0.0000000	0.0000001
52	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0.0004725	0.0000000	-0.0000006	-0.0000041	0.0000000	0.0000003
53	0	0	0	0	0	0	0	0	0	-2	0	2	0	2	-0.0003075	0.0000000	-0.0000002	0.0001313	0.0000000	-0.0000001
54	0	0	0	0	0	0	0	0	0	3	0	2	0	2	-0.0002904	0.0000000	0.0000015	0.0001233	0.0000000	0.0000007
55	0	0	0	0	0	0	0	0	0	0	-1	0	2	0	0.0004348	0.0000000	-0.0000010	-0.0000081	0.0000000	0.0000002
56	0	0	0	0	0	0	0	0	0	1	-1	2	0	2	-0.0002878	0.0000000	0.0000008	0.0001232	0.0000000	0.0000004
57	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-0.0004230	0.0000000	0.0000005	-0.0000020	0.0000000	-0.0000002
58	0	0	0	0	0	0	0	0	0	-1	-1	2	2	2	-0.0002819	0.0000000	0.0000007	0.0001207	0.0000000	0.0000003
59	0	0	0	0	0	0	0	0	0	-1	0	2	0	0	-0.0004056	0.0000000	0.0000005	0.0000040	0.0000000	-0.0000002
60	0	0	0	0	0	0	0	0	0	0	-1	2	2	2	-0.0002647	0.0000000	0.0000011	0.0001129	0.0000000	0.0000005
61	0	0	0	0	0	0	0	0	0	-2	0	0	0	1	-0.0002294	0.0000000	-0.0000010	0.0001266	0.0000000	-0.0000004
62	0	0	0	0	0	0	0	0	0	1	1	2	0	2	0.0002481	0.0000000	-0.0000007	-0.0001062	0.0000000	-0.0000003
63	0	0	0	0	0	0	0	0	0	2	0	0	0	1	0.0002179	0.0000000	-0.0000002	-0.0001129	0.0000000	-0.0000002
64	0	0	0	0	0	0	0	0	0	-1	1	0	1	0	0.0003276	0.0000000	0.0000001	-0.0000009	0.0000000	0.0000000
65	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-0.0003389	0.0000000	0.0000005	0.0000035	0.0000000	-0.0000002
66	0	0	0	0	0	0	0	0	0	1	0	2	0	0	0.0003339	0.0000000	-0.0000013	-0.0000107	0.0000000	0.0000001
67	0	0	0	0	0	0	0	0	0	-1	0	2	-2	1	-0.0001987	0.0000000	-0.0000006	0.0001073	0.0000000	-0.0000002
68	0	0	0	0	0	0	0	0	0	1	0	0	0	2	-0.0001981	0.0000000	0.0000000	0.0000854	0.0000000	0.0000000
69	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0.0004026	0.0000000	-0.0000353	-0.0000553	0.0000000	-0.0000139
70	0	0	0	0	0	0	0	0	0	0	0	2	1	2	0.0001660	0.0000000	-0.0000005	-0.0000710	0.0000000	-0.0000002
71	0	0	0	0	0	0	0	0	0	-1	0	2	4	2	-0.0001521	0.0000000	0.0000009	0.0000647	0.0000000	0.0000004
72	0	0	0	0	0	0	0	0	0	-1	1	0	1	1	0.0001314	0.0000000	0.0000000	-0.0000700	0.0000000	0.0000000
73	0	0	0	0	0	0	0	0	0	0	-2	2	-2	1	-0.0001283	0.0000000	0.0000000	0.0000672	0.0000000	0.0000000
74	0	0	0	0	0	0	0	0	0	1	0	2	2	1	-0.0001331	0.0000000	0.0000008	0.0000663	0.0000000	0.0000004
75	0	0	0	0	0	0	0	0	0	-2	0	2	2	2	0.0001383	0.0000000	-0.0000002	-0.0000594	0.0000000	-0.0000002
76	0	0	0	0	0	0	0	0	0	-1	0	0	0	2	0.0001405	0.0000000	0.0000004	-0.0000610	0.0000000	0.0000002
77	0	0	0	0	0	0	0	0	0	1	1	2	-2	2	0.0001290	0.0000000	0.0000000	-0.0000556	0.0000000	0.0000000
78	0	0	0	0	0	0	0	0	0	-2	0	2	4	2	-0.0001214	0.0000000	0.0000005	0.0000518	0.0000000	0.0000002
79	0	0	0	0	0	0	0	0	0	-1	0	4	0	2	0.0001146	0.0000000	-0.0000003	-0.0000490	0.0000000	-0.0000001
80	0	0	0	0	0	0	0	0	0	2	0	2	-2	1	0.0001019	0.0000000	-0.0000001	-0.0000527	0.0000000	-0.0000001
81	0	0	0	0	0	0	0	0	0	2	0	2	2	2	-0.0001100	0.0000000	0.0000009	0.0000465	0.0000000	0.0000004
82	0	0	0	0	0	0	0	0	0	1	0	0	2	1	-0.0000970	0.0000000	0.0000002	0.0000496	0.0000000	0.0000001
83	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0.0001575	0.0000000	-0.0000006	-0.0000050	0.0000000	0.0000000
84	0	0	0	0	0	0	0	0	0	3	0	2	-2	2	0.0000934	0.0000000	-0.0000003	-0.0000399	0.0000000	-0.0000001
85	0	0	0	0	0	0	0	0	0	0	0	4	-2	2	0.0000922	0.0000000	-0.0000001	-0.0000395	0.0000000	-0.0000001
86	0	0	0	0	0	0	0	0	0	0	1	2	0	1	0.0000815	0.0000000	-0.0000001	-0.0000422	0.0000000	-0.0000001
87	0	0	0	0	0	0	0	0	0	0	0	-2	2	1	0.0000834	0.0000000	0.0000002	-0.0000440	0.0000000	0.0000001
88	0	0	0	0	0	0	0	0	0	0	0	2	-2	3	0.0001248	0.0000000	0.0000000	-0.0000170	0.0000000	0.0000001
89	0	0	0	0	0	0	0	0	0	-1	0	0	4	0	0.0001338	0.0000000	-0.0000005	-0.0000039	0.0000000	0.0000000
90	0	0	0	0	0	0	0	0	0	2	0	-2	0	1	0.0000716	0.0000000	-0.0000002	-0.0000389	0.0000000	-0.0000001
91	0	0	0	0	0	0	0	0	0	-2	0	0	4	0	0.0001282	0.0000000	-0.0000003	-0.0000023	0.0000000	0.0000001
92	0	0	0	0	0	0	0	0	0	-1	-1	0	2	1	0.0000742	0.0000000	0.0000001	-0.0000391	0.0000000	0.0000000
93	0	0	0	0	0	0	0	0	0	-1	0	0	1	1	0.0001020	0.0000000	-0.0000025	-0.0000495	0.0000000	-0.0000010
94	0	0	0	0	0	0	0	0	0	0	1	0	0	2	0.0000715	0.0000000	-0.0000004	-0.0000326	0.0000000	0.0000002
95	0	0	0	0	0	0	0	0	0	0	0	-2	0	1	-0.0000666	0.0000000	-0.0000003	0.0000369	0.0000000	-0.0000001
96	0	0	0	0	0	0	0	0	0	0	-1	2	0	1	-0.0000667	0.0000000	0.0000001	0.0000346	0.0000000	0.0000001
97	0	0	0	0	0	0	0	0	0	0	0	2	-1	2	-0.0000704	0.0000000	0.0000000	0.0000304	0.0000000	0.0000000
98	0	0	0	0	0	0	0	0	0	0	0	2	4	2	-0.0000694	0.0000000	0.0000005	0.0000294	0.0000000	0.0000002
99	0	0	0	0	0	0	0	0	0	-2	-1	0	2	0	-0.0001014	0.0000000	-0.0000001	0.0000004	0.0000000	-0.0000001
100	0	0	0	0	0	0	0	0	0	1	1	0	-2	1	-0.0000585	0.0000000	-0.0000002	0.0000316	0.0000000	-0.0000001
101	0	0	0	0	0	0	0	0	0	-1	1	0	2	0	-0.0000949	0.0000000	0.0000001	0.0000008	0.0000000	-0.0000001
102	0	0	0	0	0	0	0	0	0	-1	1	0	1	2	-0.0000595	0.0000000	0.0000000	0.0000258	0.0000000	0.0000000
103	0	0	0	0	0	0	0	0	0	1	-1	0	0	1	0.0000528	0.0000000	0.0000000	-0.0000279	0.0000000	0.0000000
104	0	0	0	0	0	0	0	0	0	1	-1	2	2	2	-0.0000590	0.0000000	0.0000004	0.0000252	0.0000000	0.0000002
105	0	0	0	0	0	0	0	0	0	-1	1	2	2	2	0.0000570	0.0000000	-0.0000002	-0.0000244	0.0000000	-0.0000001
106	0	0	0	0	0	0	0	0	0	3	0	2	0	1	-0.0000502	0.0000000	0.0000003	0.0000250	0.0000000	0.0000002
107	0	0	0	0	0	0	0	0	0	0	1	-2	2	0	-0.0000875	0.0000000	0.0000001	0.0000029	0.0000000	0.0000000
108	0	0	0	0	0	0	0	0	0	-1	0	0	-2	1	-0.0000492	0.0000000	-0.0000003	0.0000275	0.0000000	-0.0000001
109	0	0	0	0	0	0	0	0	0	0	1	2	2	2	0.0000535	0.0000000	-0.0000002	-0.0000228	0.0000000	-0.0000001
110	0	0	0	0	0	0	0	0	0	-1	-1	2	2	1	-0.0000467	0.0000000	0.0000001	0.0000240	0.0000000	0.0000001
111	0	0	0	0	0	0	0	0	0	0	-1	0	0	2	0.0000591	0.0000000	0.0000000	-0.0000253	0.0000000	0.0000000
112	0	0	0	0	0	0	0	0	0	1	0	2	-4	1	-0.0000453	0.0000000	-0.0000001	0.0000244	0.0000000	-0.0000001
113	0	0	0	0	0	0	0	0	0	-1	0	-2	2	0	0.0000766	0.0000000	0.0000001	0.0000009	0.0000000	0.0000000
114	0	0	0	0	0	0	0	0	0	0	-1	2	2	1	-0.0000446	0.0000000	0.0000002	0.0000225	0.0000000	0.0000001
115	0	0	0	0	0	0	0	0	0	2	-1	2	0	2	-0.0000488	0.0000000	0.0000002	0.0000207	0.0000000	0.0000001
116	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-0.0000468	0.0000000	0.0000000	0.0000201	0.0000000	0.0000000
117	0	0	0	0	0	0	0	0	0	1	-1	2	0	1	-0.0000421	0.0000000	0.0000001	0.0000216	0.0000000	0.0000001
118	0	0	0	0	0	0	0	0	0	-1	1	2	0	2	0.0000463	0.0000000	0.0000000	-0.0000200	0.0000000	0.0000000
119	0	0	0	0	0	0	0	0	0	0	1	0	2	0	-0.0000673	0.0000000	0.0000002	0.0000014	0.0000000	0.0000000
120	0	0	0	0	0	0	0	0	0	0	-1	-2	2	0	0.0000658	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
121	0	0	0	0	0	0	0	0	0	0	3	2	-2	2	-0.0000438	0.0000000	0.0000000	0.0000188	0.0000000	0.0000000
122	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-0.0000390	0.0000000	0.0000000	0.0000205	0.0000000	0.0000000
123	0	0	0	0	0	0	0	0	0	-1	0	2	2	0	0.0000639	-0.0000011	-0.0000002	-0.0000019	0.0000000	0.0000000
124	0	0	0	0	0	0	0	0	0	2	1	2	0	2	0.0000412	0.0000000	-0.0000002	-0.0000176	0.0000000	-0.0000001
125	0	0	0	0	0	0	0	0	0	1	1	0	0	1	-0.0000361	0.0000000	0.0000000	0.0000189	0.0000000	0.0000000
126	0	0	0	0	0	0	0	0	0	1	1	2	0	1	0.0000360	0.0000000	-0.0000001	-0.0000185	0.0000000	-0.0000001
127	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0.0000588	0.0000000	-0.0000003	-0.0000024	0.0000000	0.0000000
128	0	0	0	0	0	0	0	0	0	1	0	-2	2	0	-0.0000578	0.0000000	0.0000001	0.0000005	0.0000000	0.0000000
129	0	0	0	0	0	0	0	0	0	-1	0	0	2	2	-0.0000396	0.0000000	0.0000000	0.0000171	0.0000000	0.0000000
130	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0.0000565	0.0000000	-0.0000001	-0.0000006	0.0000000	0.0000000
131	0	0	0	0	0	0	0	0	0	0	1	0	-2	1	-0.0000335	0.0000000	-0.0000001	0.0000184	0.0000000	-0.0000001
132	0	0	0	0	0	0	0	0	0	-1	0	2	-2	2	0.0000357	0.0000000	0.0000001	-0.0000154	0.0000000	0.0000000
133	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0.0000321	0.0000000	0.0000001	-0.0000174	0.0000000	0.0000000
134	0	0	0	0	0	0	0	0	0	-1	1	0	0	1	-0.0000301	0.0000000	-0.0000001	0.0000162	0.0000000	0.0000000
135	0	0	0	0	0	0	0	0	0	1	0	2	-1	2	-0.0000334	0.0000000	0.0000000	0.0000144	0.0000000	0.0000000
136	0	0	0	0	0	0	0	0	0	1	-1	0	2	0	0.0000493	0.0000000	-0.0000002	-0.0000015	0.0000000	0.0000000
137	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0.0000494	0.0000000	-0.0000002	-0.0000019	0.0000000	0.0000000
138	0	0	0	0	0	0	0	0	0	1	0	2	1	2	0.0000337	0.0000000	-0.0000001	-0.0000143	0.0000000	-0.0000001
139	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0.0000280	0.0000000	-0.0000001	-0.0000144	0.0000000	0.0000000
140	0	0	0	0	0	0	0	0	0	1	0	0	-2	2	0.0000309	0.0000000	0.0000001	-0.0000134	0.0000000	0.0000000
141	0	0	0	0	0	0	0	0	0	-1	0	2	4	1	-0.0000263	0.0000000	0.0000002	0.0000131	0.0000000	0.0000001
142	0	0	0	0	0	0	0	0	0	1	0	-2	0	1	0.0000253	0.0000000	0.0000001	-0.0000138	0.0000000	0.0000000
143	0	0	0	0	0	0	0	0	0	1	1	2	-2	1	0.0000245	0.0000000	0.0000000	-0.0000128	0.0000000	0.0000000
144	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0.0000416	0.0000000	-0.0000002	-0.0000017	0.0000000	0.0000000
145	0	0	0	0	0	0	0	0	0	-1	0	2	-1	1	-0.0000229	0.0000000	0.0000000	0.0000128	0.0000000	0.0000000
146	0	0	0	0	0	0	0	0	0	-2	0	2	2	1	0.0000231	0.0000000	0.0000000	-0.0000120	0.0000000	0.0000000
147	0	0	0	0	0	0	0	0	0	4	0	2	0	2	-0.0000259	0.0000000	0.0000002	0.0000109	0.0000000	0.0000001
148	0	0	0	0	0	0	0	0	0	2	-1	0	0	0	0.0000375	0.0000000	-0.0000001	-0.0000008	0.0000000	0.0000000
149	0	0	0	0	0	0	0	0	0	2	1	2	-2	2	0.0000252	0.0000000	0.0000000	-0.0000108	0.0000000	0.0000000
150	0	0	0	0	0	0	0	0	0	0	1	2	1	2	-0.0000245	0.0000000	0.0000001	0.0000104	0.0000000	0.0000000
151	0	0	0	0	0	0	0	0	0	1	0	4	-2	2	0.0000243	0.0000000	-0.0000001	-0.0000104	0.0000000	0.0000000
152	0	0	0	0	0	0	0	0	0	-1	-1	0	0	1	0.0000208	0.0000000	0.0000001	-0.0000112	0.0000000	0.0000000
153	0	0	0	0	0	0	0	0	0	0	1	0	2	1	0.0000199	0.0000000	0.0000000	-0.0000102	0.0000000	0.0000000
154	0	0	0	0	0	0	0	0	0	-2	0	2	4	1	-0.0000208	0.0000000	0.0000001	0.0000105	0.0000000	0.0000000
155	0	0	0	0	0	0	0	0	0	2	0	2	0	0	0.0000335	0.0000000	-0.0000002	-0.0000014	0.0000000	0.0000000
156	0	0	0	0	0	0	0	0	0	1	0	0	1	0	-0.0000325	0.0000000	0.0000001	0.0000007	0.0000000	0.0000000
157	0	0	0	0	0	0	0	0	0	-1	0	0	4	1	-0.0000187	0.0000000	0.0000000	0.0000096	0.0000000	0.0000000
158	0	0	0	0	0	0	0	0	0	-1	0	4	0	1	0.0000197	0.0000000	-0.0000001	-0.0000100	0.0000000	0.0000000
159	0	0	0	0	0	0	0	0	0	2	0	2	2	1	-0.0000192	0.0000000	0.0000002	0.0000094	0.0000000	0.0000001
160	0	0	0	0	0	0	0	0	0	0	0	2	-3	2	-0.0000188	0.0000000	0.0000000	0.0000083	0.0000000	0.0000000
161	0	0	0	0	0	0	0	0	0	-1	-2	0	2	0	0.0000276	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
162	0	0	0	0	0	0	0	0	0	2	1	0	0	0	-0.0000286	0.0000000	0.0000001	0.0000006	0.0000000	0.0000000
163	0	0	0	0	0	0	0	0	0	0	0	4	0	2	0.0000186	0.0000000	-0.0000001	-0.0000079	0.0000000	0.0000000
164	0	0	0	0	0	0	0	0	0	0	0	0	0	3	-0.0000219	0.0000000	0.0000000	0.0000043	0.0000000	0.0000000
165	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0.0000276	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
166	0	0	0	0	0	0	0	0	0	0	0	2	-4	1	-0.0000153	0.0000000	-0.0000001	0.0000084	0.0000000	0.0000000
167	0	0	0	0	0	0	0	0	0	0	-1	0	2	1	-0.0000156	0.0000000	0.0000000	0.0000081	0.0000000	0.0000000
168	0	0	0	0	0	0	0	0	0	0	0	0	4	1	-0.0000154	0.0000000	0.0000001	0.0000078	0.0000000	0.0000000
169	0	0	0	0	0	0	0	0	0	-1	-1	2	4	2	-0.0000174	0.0000000	0.0000001	0.0000075	0.0000000	0.0000000
170	0	0	0	0	0	0	0	0	0	1	0	2	4	2	-0.0000163	0.0000000	0.0000002	0.0000069	0.0000000	0.0000001
171	0	0	0	0	0	0	0	0	0	-2	2	0	2	0	-0.0000228	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
172	0	0	0	0	0	0	0	0	0	-2	-1	2	0	1	0.0000091	0.0000000	-0.0000004	-0.0000054	0.0000000	-0.0000002
173	0	0	0	0	0	0	0	0	0	-2	0	0	2	2	0.0000175	0.0000000	0.0000000	-0.0000075	0.0000000	0.0000000
174	0	0	0	0	0	0	0	0	0	-1	-1	2	0	2	-0.0000159	0.0000000	0.0000000	0.0000069	0.0000000	0.0000000
175	0	0	0	0	0	0	0	0	0	0	0	4	-2	1	0.0000141	0.0000000	0.0000000	-0.0000072	0.0000000	0.0000000
176	0	0	0	0	0	0	0	0	0	3	0	2	-2	1	0.0000147	0.0000000	0.0000000	-0.0000075	0.0000000	0.0000000
177	0	0	0	0	0	0	0	0	0	-2	-1	0	2	1	-0.0000132	0.0000000	0.0000000	0.0000069	0.0000000	0.0000000
178	0	0	0	0	0	0	0	0	0	1	0	0	-1	1	0.0000159	0.0000000	-0.0000028	-0.0000054	0.0000000	0.0000011
179	0	0	0	0	0	0	0	0	0	0	-2	0	2	0	0.0000213	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
180	0	0	0	0	0	0	0	0	0	-2	0	0	4	1	0.0000123	0.0000000	0.0000000	-0.0000064	0.0000000	0.0000000
181	0	0	0	0	0	0	0	0	0	-3	0	0	0	1	-0.0000118	0.0000000	-0.0000001	0.0000066	0.0000000	0.0000000
182	0	0	0	0	0	0	0	0	0	1	1	2	2	2	0.0000144	0.0000000	-0.0000001	-0.0000061	0.0000000	0.0000000
183	0	0	0	0	0	0	0	0	0	0	0	2	4	1	-0.0000121	0.0000000	0.0000001	0.0000060	0.0000000	0.0000000
184	0	0	0	0	0	0	0	0	0	3	0	2	2	2	-0.0000134	0.0000000	0.0000001	0.0000056	0.0000000	0.0000001
185	0	0	0	0	0	0	0	0	0	-1	1	2	-2	1	-0.0000105	0.0000000	0.0000000	0.0000057	0.0000000	0.0000000
186	0	0	0	0	0	0	0	0	0	2	0	0	-4	1	-0.0000102	0.0000000	0.0000000	0.0000056	0.0000000	0.0000000
187	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0.0000120	0.0000000	0.0000000	-0.0000052	0.0000000	0.0000000
188	0	0	0	0	0	0	0	0	0	2	0	2	-4	1	0.0000101	0.0000000	0.0000000	-0.0000054	0.0000000	0.0000000
189	0	0	0	0	0	0	0	0	0	-1	1	0	2	1	-0.0000113	0.0000000	0.0000000	0.0000059	0.0000000	0.0000000
190	0	0	0	0	0	0	0	0	0	0	0	2	-1	1	-0.0000106	0.0000000	0.0000000	0.0000061	0.0000000	0.0000000
191	0	0	0	0	0	0	0	0	0	0	-2	2	2	2	-0.0000129	0.0000000	0.0000001	0.0000055	0.0000000	0.0000000
192	0	0	0	0	0	0	0	0	0	2	0	0	2	1	-0.0000114	0.0000000	0.0000000	0.0000057	0.0000000	0.0000000
193	0	0	0	0	0	0	0	0	0	4	0	2	-2	2	0.0000113	0.0000000	-0.0000001	-0.0000049	0.0000000	0.0000000
194	0	0	0	0	0	0	0	0	0	2	0	0	-2	2	-0.0000102	0.0000000	0.0000000	0.0000044	0.0000000	0.0000000
195	0	0	0	0	0	0	0	0	0	0	2	0	0	1	-0.0000094	0.0000000	0.0000000	0.0000051	0.0000000	0.0000000
196	0	0	0	0	0	0	0	0	0	1	0	0	-4	1	-0.0000100	0.0000000	-0.0000001	0.0000056	0.0000000	0.0000000
197	0	0	0	0	0	0	0	0	0	0	2	2	-2	1	0.0000087	0.0000000	0.0000000	-0.0000047	0.0000000	0.0000000
198	0	0	0	0	0	0	0	0	0	-3	0	0	4	0	0.0000161	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
199	0	0	0	0	0	0	0	0	0	-1	1	2	0	1	0.0000096	0.0000000	0.0000000	-0.0000050	0.0000000	0.0000000
200	0	0	0	0	0	0	0	0	0	-1	-1	0	4	0	0.0000151	0.0000000	-0.0000001	-0.0000005	0.0000000	0.0000000
201	0	0	0	0	0	0	0	0	0	-1	-2	2	2	2	-0.0000104	0.0000000	0.0000000	0.0000044	0.0000000	0.0000000
202	0	0	0	0	0	0	0	0	0	-2	-1	2	4	2	-0.0000110	0.0000000	0.0000000	0.0000048	0.0000000	0.0000000
203	0	0	0	0	0	0	0	0	0	1	-1	2	2	1	-0.0000100	0.0000000	0.0000001	0.0000050	0.0000000	0.0000000
204	0	0	0	0	0	0	0	0	0	-2	1	0	2	0	0.0000092	0.0000000	-0.0000005	0.0000012	0.0000000	-0.0000002
205	0	0	0	0	0	0	0	0	0	-2	1	2	0	1	0.0000082	0.0000000	0.0000000	-0.0000045	0.0000000	0.0000000
206	0	0	0	0	0	0	0	0	0	2	1	0	-2	1	0.0000082	0.0000000	0.0000000	-0.0000045	0.0000000	0.0000000
207	0	0	0	0	0	0	0	0	0	-3	0	2	0	1	-0.0000078	0.0000000	0.0000000	0.0000041	0.0000000	0.0000000
208	0	0	0	0	0	0	0	0	0	-2	0	2	-2	1	-0.0000077	0.0000000	0.0000000	0.0000043	0.0000000	0.0000000
209	0	0	0	0	0	0	0	0	0	-1	1	0	2	2	0.0000002	0.0000000	0.0000000	0.0000054	0.0000000	0.0000000
210	0	0	0	0	0	0	0	0	0	0	-1	2	-1	2	0.0000094	0.0000000	0.0000000	-0.0000040	0.0000000	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														Δψ Coefficients			Δε Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
211	0	0	0	0	0	0	0	0	0	-1	0	4	-2	2	-0.0000093	0.0000000	0.0000000	0.0000040	0.0000000	0.0000000
212	0	0	0	0	0	0	0	0	0	0	-2	2	0	2	-0.0000083	0.0000000	0.0000010	0.0000040	0.0000000	-0.0000002
213	0	0	0	0	0	0	0	0	0	-1	0	2	1	2	0.0000083	0.0000000	0.0000000	-0.0000036	0.0000000	0.0000000
214	0	0	0	0	0	0	0	0	0	2	0	0	0	2	-0.0000091	0.0000000	0.0000000	0.0000039	0.0000000	0.0000000
215	0	0	0	0	0	0	0	0	0	0	0	2	0	3	0.0000128	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
216	0	0	0	0	0	0	0	0	0	-2	0	4	0	2	-0.0000079	0.0000000	0.0000000	0.0000034	0.0000000	0.0000000
217	0	0	0	0	0	0	0	0	0	-1	0	-2	0	1	-0.0000083	0.0000000	0.0000000	0.0000047	0.0000000	0.0000000
218	0	0	0	0	0	0	0	0	0	-1	1	2	2	1	0.0000084	0.0000000	0.0000000	-0.0000044	0.0000000	0.0000000
219	0	0	0	0	0	0	0	0	0	3	0	0	0	1	0.0000083	0.0000000	0.0000000	-0.0000043	0.0000000	0.0000000
220	0	0	0	0	0	0	0	0	0	-1	0	2	3	2	0.0000091	0.0000000	0.0000000	-0.0000039	0.0000000	0.0000000
221	0	0	0	0	0	0	0	0	0	2	-1	2	0	1	-0.0000077	0.0000000	0.0000000	0.0000039	0.0000000	0.0000000
222	0	0	0	0	0	0	0	0	0	0	1	2	2	1	0.0000084	0.0000000	0.0000000	-0.0000043	0.0000000	0.0000000
223	0	0	0	0	0	0	0	0	0	0	-1	2	4	2	-0.0000092	0.0000000	0.0000001	0.0000039	0.0000000	0.0000000
224	0	0	0	0	0	0	0	0	0	2	-1	2	2	2	-0.0000092	0.0000000	0.0000001	0.0000039	0.0000000	0.0000000
225	0	0	0	0	0	0	0	0	0	0	2	-2	2	0	-0.0000094	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
226	0	0	0	0	0	0	0	0	0	-1	-1	2	-1	1	0.0000068	0.0000000	0.0000000	-0.0000036	0.0000000	0.0000000
227	0	0	0	0	0	0	0	0	0	0	-2	0	0	1	-0.0000061	0.0000000	0.0000000	0.0000032	0.0000000	0.0000000
228	0	0	0	0	0	0	0	0	0	1	0	2	-4	2	0.0000071	0.0000000	0.0000000	-0.0000031	0.0000000	0.0000000
229	0	0	0	0	0	0	0	0	0	1	-1	0	-2	1	0.0000062	0.0000000	0.0000000	-0.0000034	0.0000000	0.0000000
230	0	0	0	0	0	0	0	0	0	-1	-1	2	0	1	-0.0000063	0.0000000	0.0000000	0.0000033	0.0000000	0.0000000
231	0	0	0	0	0	0	0	0	0	1	-1	2	-2	2	-0.0000073	0.0000000	0.0000000	0.0000032	0.0000000	0.0000000
232	0	0	0	0	0	0	0	0	0	-2	-1	0	4	0	0.0000115	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
233	0	0	0	0	0	0	0	0	0	-1	0	0	3	0	-0.0000103	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
234	0	0	0	0	0	0	0	0	0	-2	-1	2	2	2	0.0000063	0.0000000	0.0000000	-0.0000028	0.0000000	0.0000000
235	0	0	0	0	0	0	0	0	0	0	2	2	0	2	0.0000074	0.0000000	0.0000000	-0.0000032	0.0000000	0.0000000
236	0	0	0	0	0	0	0	0	0	1	1	0	2	0	-0.0000103	0.0000000	-0.0000003	0.0000003	0.0000000	-0.0000001
237	0	0	0	0	0	0	0	0	0	2	0	2	-1	2	-0.0000069	0.0000000	0.0000000	0.0000030	0.0000000	0.0000000
238	0	0	0	0	0	0	0	0	0	1	0	2	1	1	0.0000057	0.0000000	0.0000000	-0.0000029	0.0000000	0.0000000
239	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0.0000094	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
240	0	0	0	0	0	0	0	0	0	2	1	2	0	1	0.0000064	0.0000000	0.0000000	-0.0000033	0.0000000	0.0000000
241	0	0	0	0	0	0	0	0	0	3	-1	2	0	2	-0.0000063	0.0000000	0.0000000	0.0000026	0.0000000	0.0000000
242	0	0	0	0	0	0	0	0	0	-2	2	0	2	1	-0.0000038	0.0000000	0.0000000	0.0000020	0.0000000	0.0000000
243	0	0	0	0	0	0	0	0	0	1	0	2	-3	1	-0.0000043	0.0000000	0.0000000	0.0000024	0.0000000	0.0000000
244	0	0	0	0	0	0	0	0	0	1	1	2	-4	1	-0.0000045	0.0000000	0.0000000	0.0000023	0.0000000	0.0000000
245	0	0	0	0	0	0	0	0	0	-1	-1	2	-2	1	0.0000047	0.0000000	0.0000000	-0.0000024	0.0000000	0.0000000
246	0	0	0	0	0	0	0	0	0	0	-1	0	-1	1	-0.0000048	0.0000000	0.0000000	0.0000025	0.0000000	0.0000000
247	0	0	0	0	0	0	0	0	0	0	-1	0	-2	1	0.0000045	0.0000000	0.0000000	-0.0000026	0.0000000	0.0000000
248	0	0	0	0	0	0	0	0	0	-2	0	0	0	2	0.0000056	0.0000000	0.0000000	-0.0000025	0.0000000	0.0000000
249	0	0	0	0	0	0	0	0	0	-2	0	-2	2	0	0.0000088	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
250	0	0	0	0	0	0	0	0	0	-1	0	-2	4	0	-0.0000075	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
251	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0.0000085	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
252	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0.0000049	0.0000000	0.0000000	-0.0000026	0.0000000	0.0000000
253	0	0	0	0	0	0	0	0	0	-1	2	0	2	0	-0.0000074	0.0000000	-0.0000003	-0.0000001	0.0000000	-0.0000001
254	0	0	0	0	0	0	0	0	0	1	-1	2	-2	1	-0.0000039	0.0000000	0.0000000	0.0000021	0.0000000	0.0000000
255	0	0	0	0	0	0	0	0	0	1	2	2	-2	2	0.0000045	0.0000000	0.0000000	-0.0000020	0.0000000	0.0000000
256	0	0	0	0	0	0	0	0	0	2	-1	2	-2	2	0.0000051	0.0000000	0.0000000	-0.0000022	0.0000000	0.0000000
257	0	0	0	0	0	0	0	0	0	1	0	2	-1	1	-0.0000040	0.0000000	0.0000000	0.0000021	0.0000000	0.0000000
258	0	0	0	0	0	0	0	0	0	2	1	2	-2	1	0.0000041	0.0000000	0.0000000	-0.0000021	0.0000000	0.0000000
259	0	0	0	0	0	0	0	0	0	-2	0	0	-2	1	-0.0000042	0.0000000	0.0000000	0.0000024	0.0000000	0.0000000
260	0	0	0	0	0	0	0	0	0	1	-2	2	0	2	-0.0000051	0.0000000	0.0000000	0.0000022	0.0000000	0.0000000
261	0	0	0	0	0	0	0	0	0	0	1	2	1	1	-0.0000042	0.0000000	0.0000000	0.0000022	0.0000000	0.0000000
262	0	0	0	0	0	0	0	0	0	1	0	4	-2	1	0.0000039	0.0000000	0.0000000	-0.0000021	0.0000000	0.0000000
263	0	0	0	0	0	0	0	0	0	-2	0	4	2	2	0.0000046	0.0000000	0.0000000	-0.0000018	0.0000000	0.0000000
264	0	0	0	0	0	0	0	0	0	1	1	2	1	2	-0.0000053	0.0000000	0.0000000	0.0000022	0.0000000	0.0000000
265	0	0	0	0	0	0	0	0	0	1	0	0	4	0	0.0000082	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
266	0	0	0	0	0	0	0	0	0	1	0	2	2	0	0.0000081	0.0000000	-0.0000001	-0.0000004	0.0000000	0.0000000
267	0	0	0	0	0	0	0	0	0	2	0	2	1	2	0.0000047	0.0000000	0.0000000	-0.0000019	0.0000000	0.0000000
268	0	0	0	0	0	0	0	0	0	3	1	2	0	2	0.0000053	0.0000000	0.0000000	-0.0000023	0.0000000	0.0000000
269	0	0	0	0	0	0	0	0	0	4	0	2	0	1	-0.0000045	0.0000000	0.0000000	0.0000022	0.0000000	0.0000000
270	0	0	0	0	0	0	0	0	0	-2	-1	2	0	0	-0.0000044	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
271	0	0	0	0	0	0	0	0	0	0	1	-2	2	1	-0.0000033	0.0000000	0.0000000	0.0000016	0.0000000	0.0000000
272	0	0	0	0	0	0	0	0	0	1	0	-2	1	0	-0.0000061	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
273	0	0	0	0	0	0	0	0	0	0	-1	-2	2	1	0.0000028	0.0000000	0.0000000	-0.0000015	0.0000000	0.0000000
274	0	0	0	0	0	0	0	0	0	2	-1	0	-2	1	-0.0000038	0.0000000	0.0000000	0.0000019	0.0000000	0.0000000
275	0	0	0	0	0	0	0	0	0	-1	0	2	-1	2	-0.0000033	0.0000000	0.0000000	0.0000021	0.0000000	0.0000000
276	0	0	0	0	0	0	0	0	0	1	0	2	-3	2	-0.0000060	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
277	0	0	0	0	0	0	0	0	0	0	1	2	-2	3	0.0000048	0.0000000	0.0000000	-0.0000010	0.0000000	0.0000000
278	0	0	0	0	0	0	0	0	0	0	0	2	-3	1	0.0000027	0.0000000	0.0000000	-0.0000014	0.0000000	0.0000000
279	0	0	0	0	0	0	0	0	0	-1	0	-2	2	1	0.0000038	0.0000000	0.0000000	-0.0000020	0.0000000	0.0000000
280	0	0	0	0	0	0	0	0	0	0	0	2	-4	2	0.0000031	0.0000000	0.0000000	-0.0000013	0.0000000	0.0000000
281	0	0	0	0	0	0	0	0	0	-2	1	0	0	1	-0.0000029	0.0000000	0.0000000	0.0000015	0.0000000	0.0000000
282	0	0	0	0	0	0	0	0	0	-1	0	0	-1	1	0.0000028	0.0000000	0.0000000	-0.0000015	0.0000000	0.0000000
283	0	0	0	0	0	0	0	0	0	2	0	2	-4	2	-0.0000032	0.0000000	0.0000000	0.0000015	0.0000000	0.0000000
284	0	0	0	0	0	0	0	0	0	0	0	4	-4	4	0.0000045	0.0000000	0.0000000	-0.0000008	0.0000000	0.0000000
285	0	0	0	0	0	0	0	0	0	0	0	4	-4	2	-0.0000044	0.0000000	0.0000000	0.0000019	0.0000000	0.0000000
286	0	0	0	0	0	0	0	0	0	-1	-2	0	2	1	0.0000028	0.0000000	0.0000000	-0.0000015	0.0000000	0.0000000
287	0	0	0	0	0	0	0	0	0	-2	0	0	3	0	-0.0000051	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
288	0	0	0	0	0	0	0	0	0	1	0	-2	2	1	-0.0000036	0.0000000	0.0000000	0.0000020	0.0000000	0.0000000
289	0	0	0	0	0	0	0	0	0	-3	0	2	2	2	0.0000044	0.0000000	0.0000000	-0.0000019	0.0000000	0.0000000
290	0	0	0	0	0	0	0	0	0	-3	0	2	2	1	0.0000026	0.0000000	0.0000000	-0.0000014	0.0000000	0.0000000
291	0	0	0	0	0	0	0	0	0	-2	0	2	2	0	-0.0000060	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
292	0	0	0	0	0	0	0	0	0	2	-1	0	0	1	0.0000035	0.0000000	0.0000000	-0.0000018	0.0000000	0.0000000
293	0	0	0	0	0	0	0	0	0	-2	1	2	2	2	-0.0000027	0.0000000	0.0000000	0.0000011	0.0000000	0.0000000
294	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0.0000047	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
295	0	0	0	0	0	0	0	0	0	0	1	4	-2	2	0.0000036	0.0000000	0.0000000	-0.0000015	0.0000000	0.0000000
296	0	0	0	0	0	0	0	0	0	-1	1	0	-2	1	-0.0000036	0.0000000	0.0000000	0.0000020	0.0000000	0.0000000
297	0	0	0	0	0	0	0	0	0	0	0	0	-4	1	-0.0000035	0.0000000	0.0000000	0.0000019	0.0000000	0.0000000
298	0	0	0	0	0	0	0	0	0	1	-1	0	2	1	-0.0000037	0.0000000	0.0000000	0.0000019	0.0000000	0.0000000
299	0	0	0	0	0	0	0	0	0	1	1	0	2	1	0.0000032	0.0000000	0.0000000	-0.0000016	0.0000000	0.0000000
300	0	0	0	0	0	0	0	0	0	-1	2	2	2	2	0.0000035	0.0000000	0.0000000	-0.0000014	0.0000000	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
301	0	0	0	0	0	0	0	0	0	3	1	2	-2	2	0.0000032	0.0000000	0.0000000	-0.0000013	0.0000000	0.0000000
302	0	0	0	0	0	0	0	0	0	0	-1	0	4	0	0.0000065	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
303	0	0	0	0	0	0	0	0	0	2	-1	0	2	0	0.0000047	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
304	0	0	0	0	0	0	0	0	0	0	0	4	0	1	0.0000032	0.0000000	0.0000000	-0.0000016	0.0000000	0.0000000
305	0	0	0	0	0	0	0	0	0	2	0	4	-2	2	0.0000037	0.0000000	0.0000000	-0.0000016	0.0000000	0.0000000
306	0	0	0	0	0	0	0	0	0	-1	-1	2	4	1	-0.0000030	0.0000000	0.0000000	0.0000015	0.0000000	0.0000000
307	0	0	0	0	0	0	0	0	0	1	0	0	4	1	-0.0000032	0.0000000	0.0000000	0.0000016	0.0000000	0.0000000
308	0	0	0	0	0	0	0	0	0	1	-2	2	2	2	-0.0000031	0.0000000	0.0000000	0.0000013	0.0000000	0.0000000
309	0	0	0	0	0	0	0	0	0	0	0	2	3	2	0.0000037	0.0000000	0.0000000	-0.0000016	0.0000000	0.0000000
310	0	0	0	0	0	0	0	0	0	-1	1	2	4	2	0.0000031	0.0000000	0.0000000	-0.0000013	0.0000000	0.0000000
311	0	0	0	0	0	0	0	0	0	3	0	0	2	0	0.0000049	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
312	0	0	0	0	0	0	0	0	0	-1	0	4	2	2	0.0000032	0.0000000	0.0000000	-0.0000013	0.0000000	0.0000000
313	0	0	0	0	0	0	0	0	0	1	1	2	2	1	0.0000023	0.0000000	0.0000000	-0.0000012	0.0000000	0.0000000
314	0	0	0	0	0	0	0	0	0	-2	0	2	6	2	-0.0000043	0.0000000	0.0000000	0.0000018	0.0000000	0.0000000
315	0	0	0	0	0	0	0	0	0	2	1	2	2	2	0.0000026	0.0000000	0.0000000	-0.0000011	0.0000000	0.0000000
316	0	0	0	0	0	0	0	0	0	-1	0	2	6	2	-0.0000032	0.0000000	0.0000000	0.0000014	0.0000000	0.0000000
317	0	0	0	0	0	0	0	0	0	1	0	2	4	1	-0.0000029	0.0000000	0.0000000	0.0000014	0.0000000	0.0000000
318	0	0	0	0	0	0	0	0	0	2	0	2	4	2	-0.0000027	0.0000000	0.0000000	0.0000012	0.0000000	0.0000000
319	0	0	0	0	0	0	0	0	0	1	1	-2	1	0	0.0000030	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
320	0	0	0	0	0	0	0	0	0	-3	1	2	1	2	-0.0000011	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
321	0	0	0	0	0	0	0	0	0	2	0	-2	0	2	-0.0000021	0.0000000	0.0000000	0.0000010	0.0000000	0.0000000
322	0	0	0	0	0	0	0	0	0	-1	0	0	1	2	-0.0000034	0.0000000	0.0000000	0.0000015	0.0000000	0.0000000
323	0	0	0	0	0	0	0	0	0	-4	0	2	2	1	-0.0000010	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000
324	0	0	0	0	0	0	0	0	0	-1	-1	0	1	0	-0.0000036	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
325	0	0	0	0	0	0	0	0	0	0	0	-2	2	2	-0.0000009	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
326	0	0	0	0	0	0	0	0	0	1	0	0	-1	2	-0.0000012	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
327	0	0	0	0	0	0	0	0	0	0	-1	2	-2	3	-0.0000021	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
328	0	0	0	0	0	0	0	0	0	-2	1	2	0	0	-0.0000029	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
329	0	0	0	0	0	0	0	0	0	0	0	2	-2	4	-0.0000015	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
330	0	0	0	0	0	0	0	0	0	-2	-2	0	2	0	-0.0000020	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
331	0	0	0	0	0	0	0	0	0	-2	0	-2	4	0	0.0000028	0.0000000	0.0000000	0.0000000	0.0000000	-0.0000002
332	0	0	0	0	0	0	0	0	0	0	-2	-2	2	0	0.0000017	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
333	0	0	0	0	0	0	0	0	0	1	2	0	-2	1	-0.0000022	0.0000000	0.0000000	0.0000012	0.0000000	0.0000000
334	0	0	0	0	0	0	0	0	0	3	0	0	-4	1	-0.0000014	0.0000000	0.0000000	0.0000007	0.0000000	0.0000000
335	0	0	0	0	0	0	0	0	0	-1	1	2	-2	2	0.0000024	0.0000000	0.0000000	-0.0000011	0.0000000	0.0000000
336	0	0	0	0	0	0	0	0	0	1	-1	2	-4	1	0.0000011	0.0000000	0.0000000	-0.0000006	0.0000000	0.0000000
337	0	0	0	0	0	0	0	0	0	1	1	0	-2	2	0.0000014	0.0000000	0.0000000	-0.0000006	0.0000000	0.0000000
338	0	0	0	0	0	0	0	0	0	-3	0	2	0	0	0.0000024	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
339	0	0	0	0	0	0	0	0	0	-3	0	2	0	2	0.0000018	0.0000000	0.0000000	-0.0000008	0.0000000	0.0000000
340	0	0	0	0	0	0	0	0	0	-2	0	0	1	0	-0.0000038	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
341	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	-0.0000031	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
342	0	0	0	0	0	0	0	0	0	-3	0	0	2	1	-0.0000016	0.0000000	0.0000000	0.0000008	0.0000000	0.0000000
343	0	0	0	0	0	0	0	0	0	-1	-1	-2	2	0	0.0000029	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
344	0	0	0	0	0	0	0	0	0	0	1	2	-4	1	-0.0000018	0.0000000	0.0000000	0.0000010	0.0000000	0.0000000
345	0	0	0	0	0	0	0	0	0	2	1	0	-4	1	-0.0000010	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
346	0	0	0	0	0	0	0	0	0	0	2	0	-2	1	-0.0000017	0.0000000	0.0000000	0.0000010	0.0000000	0.0000000
347	0	0	0	0	0	0	0	0	0	1	0	0	-3	1	0.0000009	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
348	0	0	0	0	0	0	0	0	0	-2	0	2	-2	2	0.0000016	0.0000000	0.0000000	-0.0000006	0.0000000	0.0000000
349	0	0	0	0	0	0	0	0	0	-2	-1	0	0	1	0.0000022	0.0000000	0.0000000	-0.0000012	0.0000000	0.0000000
350	0	0	0	0	0	0	0	0	0	-4	0	0	2	0	0.0000020	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
351	0	0	0	0	0	0	0	0	0	1	1	0	-4	1	-0.0000013	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000
352	0	0	0	0	0	0	0	0	0	-1	0	2	-4	1	-0.0000017	0.0000000	0.0000000	0.0000009	0.0000000	0.0000000
353	0	0	0	0	0	0	0	0	0	0	0	4	-4	1	-0.0000014	0.0000000	0.0000000	0.0000008	0.0000000	0.0000000
354	0	0	0	0	0	0	0	0	0	0	3	2	-2	2	0.0000000	0.0000000	0.0000000	-0.0000007	0.0000000	0.0000000
355	0	0	0	0	0	0	0	0	0	-3	-1	0	4	0	0.0000014	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
356	0	0	0	0	0	0	0	0	0	-3	0	0	4	1	0.0000019	0.0000000	0.0000000	-0.0000010	0.0000000	0.0000000
357	0	0	0	0	0	0	0	0	0	1	-1	-2	2	0	-0.0000034	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
358	0	0	0	0	0	0	0	0	0	-1	-1	0	2	2	-0.0000020	0.0000000	0.0000000	0.0000008	0.0000000	0.0000000
359	0	0	0	0	0	0	0	0	0	1	-2	0	0	1	0.0000009	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000
360	0	0	0	0	0	0	0	0	0	1	-1	0	0	2	-0.0000018	0.0000000	0.0000000	0.0000007	0.0000000	0.0000000
361	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0.0000013	0.0000000	0.0000000	-0.0000006	0.0000000	0.0000000
362	0	0	0	0	0	0	0	0	0	-1	-1	2	0	0	0.0000017	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
363	0	0	0	0	0	0	0	0	0	1	-2	2	-2	2	-0.0000012	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
364	0	0	0	0	0	0	0	0	0	0	-1	2	-1	1	0.0000015	0.0000000	0.0000000	-0.0000008	0.0000000	0.0000000
365	0	0	0	0	0	0	0	0	0	-1	0	2	0	3	-0.0000011	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
366	0	0	0	0	0	0	0	0	0	1	1	0	0	2	0.0000013	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000
367	0	0	0	0	0	0	0	0	0	-1	1	2	0	0	-0.0000018	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
368	0	0	0	0	0	0	0	0	0	1	2	0	0	0	-0.0000035	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
369	0	0	0	0	0	0	0	0	0	-1	2	2	0	2	0.0000009	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
370	0	0	0	0	0	0	0	0	0	-1	0	4	-2	1	-0.0000019	0.0000000	0.0000000	0.0000010	0.0000000	0.0000000
371	0	0	0	0	0	0	0	0	0	3	0	2	-4	2	-0.0000026	0.0000000	0.0000000	0.0000011	0.0000000	0.0000000
372	0	0	0	0	0	0	0	0	0	1	2	2	-2	1	0.0000008	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
373	0	0	0	0	0	0	0	0	0	1	0	4	-4	2	-0.0000010	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
374	0	0	0	0	0	0	0	0	0	-2	-1	0	4	1	0.0000010	0.0000000	0.0000000	-0.0000006	0.0000000	0.0000000
375	0	0	0	0	0	0	0	0	0	0	-1	0	2	2	-0.0000021	0.0000000	0.0000000	0.0000009	0.0000000	0.0000000
376	0	0	0	0	0	0	0	0	0	-2	1	0	4	0	-0.0000015	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
377	0	0	0	0	0	0	0	0	0	-2	-1	2	2	1	0.0000009	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000
378	0	0	0	0	0	0	0	0	0	2	0	-2	2	0	-0.0000029	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
379	0	0	0	0	0	0	0	0	0	1	0	0	1	1	-0.0000019	0.0000000	0.0000000	0.0000010	0.0000000	0.0000000
380	0	0	0	0	0	0	0	0	0	0	1	0	2	2	0.0000012	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000
381	0	0	0	0	0	0	0	0	0	1	-1	2	-1	2	0.0000022	0.0000000	0.0000000	-0.0000009	0.0000000	0.0000000
382	0	0	0	0	0	0	0	0	0	-2	0	4	0	1	-0.0000010	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
383	0	0	0	0	0	0	0	0	0	2	1	0	0	1	-0.0000020	0.0000000	0.0000000	0.0000011	0.0000000	0.0000000
384	0	0	0	0	0	0	0	0	0	0	1	2	0	0	-0.0000020	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
385	0	0	0	0	0	0	0	0	0	0	-1	4	-2	2	-0.0000017	0.0000000	0.0000000	0.0000007	0.0000000	0.0000000
386	0	0	0	0	0	0	0	0	0	0	0	4	-2	4	0.0000015	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
387	0	0	0	0	0	0	0	0	0	0	2	2	0	1	0.0000008	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
388	0	0	0	0	0	0	0	0	0	-3	0	0	6	0	0.0000014	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
389	0	0	0	0	0	0	0	0	0	-1	-1	0	4	1	-0.0000012	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000
390	0	0	0	0	0	0	0	0	0	1	-2	0	2	0	0.0000025	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
391	0	0	0	0	0	0	0	0	0	-1	0	0	4	2	-0.0000013	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000
392	0	0	0	0	0	0	0	0	0	-1	-2	2	2	1	-0.0000014	0.0000000	0.0000000	0.0000008	0.0000000	0.0000000
393	0	0	0	0	0	0	0	0	0	-1	0	0	-2	2	0.0000013	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000
394	0	0	0	0	0	0	0	0	0	1	0	-2	-2	1	-0.0000017	0.0000000	0.0000000	0.0000009	0.0000000	0.0000000
395	0	0	0	0	0	0	0	0	0	0	0	-2	-2	1	-0.0000012	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000
396	0	0	0	0	0	0	0	0	0	-2	0	-2	0	1	-0.0000010	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
397	0	0	0	0	0	0	0	0	0	0	0	0	3	1	0.0000010	0.0000000	0.0000000	-0.0000006	0.0000000	0.0000000
398	0	0	0	0	0	0	0	0	0	0	0	0	3	0	-0.0000015	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
399	0	0	0	0	0	0	0	0	0	-1	1	0	4	0	-0.0000022	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
400	0	0	0	0	0	0	0	0	0	-1	-1	2	2	0	0.0000028	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
401	0	0	0	0	0	0	0	0	0	-2	0	2	3	2	0.0000015	0.0000000	0.0000000	-0.0000007	0.0000000	0.0000000
402	0	0	0	0	0	0	0	0	0	1	0	0	2	2	0.0000023	0.0000000	0.0000000	-0.0000010	0.0000000	0.0000000
403	0	0	0	0	0	0	0	0	0	0	-1	2	1	2	0.0000012	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000
404	0	0	0	0	0	0	0	0	0	3	-1	0	0	0	0.0000029	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
405	0	0	0	0	0	0	0	0	0	2	0	0	1	0	-0.0000025	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
406	0	0	0	0	0	0	0	0	0	1	-1	2	0	0	0.0000022	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
407	0	0	0	0	0	0	0	0	0	0	0	2	1	0	-0.0000018	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
408	0	0	0	0	0	0	0	0	0	1	0	2	0	3	0.0000015	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
409	0	0	0	0	0	0	0	0	0	3	1	0	0	0	-0.0000023	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
410	0	0	0	0	0	0	0	0	0	3	-1	2	-2	2	0.0000012	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000
411	0	0	0	0	0	0	0	0	0	2	0	2	-1	1	-0.0000008	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
412	0	0	0	0	0	0	0	0	0	1	1	2	0	0	-0.0000019	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
413	0	0	0	0	0	0	0	0	0	0	0	4	-1	2	-0.0000010	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
414	0	0	0	0	0	0	0	0	0	1	2	2	0	2	0.0000021	0.0000000	0.0000000	-0.0000009	0.0000000	0.0000000
415	0	0	0	0	0	0	0	0	0	-2	0	0	6	0	0.0000023	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
416	0	0	0	0	0	0	0	0	0	0	-1	0	4	1	-0.0000016	0.0000000	0.0000000	0.0000008	0.0000000	0.0000000
417	0	0	0	0	0	0	0	0	0	-2	-1	2	4	1	-0.0000019	0.0000000	0.0000000	0.0000009	0.0000000	0.0000000
418	0	0	0	0	0	0	0	0	0	0	-2	2	2	1	-0.0000022	0.0000000	0.0000000	0.0000010	0.0000000	0.0000000
419	0	0	0	0	0	0	0	0	0	0	-1	2	2	0	0.0000027	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
420	0	0	0	0	0	0	0	0	0	-1	0	2	3	1	0.0000016	0.0000000	0.0000000	-0.0000008	0.0000000	0.0000000
421	0	0	0	0	0	0	0	0	0	-2	1	2	4	2	0.0000019	0.0000000	0.0000000	-0.0000008	0.0000000	0.0000000
422	0	0	0	0	0	0	0	0	0	2	0	0	2	2	0.0000009	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
423	0	0	0	0	0	0	0	0	0	2	-2	2	0	2	-0.0000009	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
424	0	0	0	0	0	0	0	0	0	-1	1	2	3	2	-0.0000009	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
425	0	0	0	0	0	0	0	0	0	3	0	2	-1	2	-0.0000008	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
426	0	0	0	0	0	0	0	0	0	4	0	2	-2	1	0.0000018	0.0000000	0.0000000	-0.0000009	0.0000000	0.0000000
427	0	0	0	0	0	0	0	0	0	-1	0	0	6	0	0.0000016	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
428	0	0	0	0	0	0	0	0	0	-1	-2	2	4	2	-0.0000010	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
429	0	0	0	0	0	0	0	0	0	-3	0	2	6	2	-0.0000023	0.0000000	0.0000000	0.0000009	0.0000000	0.0000000
430	0	0	0	0	0	0	0	0	0	-1	0	2	4	0	0.0000016	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
431	0	0	0	0	0	0	0	0	0	3	0	0	2	1	-0.0000012	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000
432	0	0	0	0	0	0	0	0	0	3	-1	2	0	1	-0.0000008	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
433	0	0	0	0	0	0	0	0	0	3	0	2	0	0	0.0000030	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
434	0	0	0	0	0	0	0	0	0	1	0	4	0	2	0.0000024	0.0000000	0.0000000	-0.0000010	0.0000000	0.0000000
435	0	0	0	0	0	0	0	0	0	5	0	2	-2	2	0.0000010	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
436	0	0	0	0	0	0	0	0	0	0	-1	2	4	1	-0.0000016	0.0000000	0.0000000	0.0000007	0.0000000	0.0000000
437	0	0	0	0	0	0	0	0	0	2	-1	2	2	1	-0.0000016	0.0000000	0.0000000	0.0000007	0.0000000	0.0000000
438	0	0	0	0	0	0	0	0	0	0	1	2	4	2	0.0000017	0.0000000	0.0000000	-0.0000007	0.0000000	0.0000000
439	0	0	0	0	0	0	0	0	0	1	-1	2	4	2	-0.0000024	0.0000000	0.0000000	0.0000010	0.0000000	0.0000000
440	0	0	0	0	0	0	0	0	0	3	-1	2	2	2	-0.0000012	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
441	0	0	0	0	0	0	0	0	0	3	0	2	2	1	-0.0000024	0.0000000	0.0000000	0.0000011	0.0000000	0.0000000
442	0	0	0	0	0	0	0	0	0	5	0	2	0	2	-0.0000023	0.0000000	0.0000000	0.0000009	0.0000000	0.0000000
443	0	0	0	0	0	0	0	0	0	0	0	2	6	2	-0.0000013	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000
444	0	0	0	0	0	0	0	0	0	4	0	2	2	2	-0.0000015	0.0000000	0.0000000	0.0000007	0.0000000	0.0000000
445	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1	0.0000000	0.0000000	-0.0001988	0.0000000	0.0000000	-0.0001679
446	0	0	0	0	0	0	0	0	0	-1	0	1	0	3	0.0000000	0.0000000	-0.0000063	0.0000000	0.0000000	-0.0000027
447	0	0	0	0	0	0	0	0	0	0	-2	2	-2	3	-0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
448	0	0	0	0	0	0	0	0	0	1	0	-1	0	1	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000	0.0000004
449	0	0	0	0	0	0	0	0	0	2	-2	0	-2	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
450	0	0	0	0	0	0	0	0	0	-1	0	1	0	2	0.0000000	0.0000000	0.0000364	0.0000000	0.0000000	0.0000176
451	0	0	0	0	0	0	0	0	0	-1	0	1	0	1	0.0000000	0.0000000	-0.0001044	0.0000000	0.0000000	-0.0000891
452	0	0	0	0	0	0	0	0	0	-1	-1	2	-1	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
453	0	0	0	0	0	0	0	0	0	-2	2	0	2	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
454	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0.0000000	0.0000000	0.0000330	0.0000000	0.0000000	0.0000000
455	0	0	0	0	0	0	0	0	0	-4	1	2	2	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
456	0	0	0	0	0	0	0	0	0	-3	0	2	1	1	0.0000003	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
457	0	0	0	0	0	0	0	0	0	-2	-1	2	0	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
458	0	0	0	0	0	0	0	0	0	1	0	-2	1	1	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
459	0	0	0	0	0	0	0	0	0	2	-1	-2	0	1	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
460	0	0	0	0	0	0	0	0	0	-4	0	2	2	0	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
461	0	0	0	0	0	0	0	0	0	-3	1	0	3	0	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
462	0	0	0	0	0	0	0	0	0	-1	0	-1	2	0	0.0000000	0.0000000	0.0000005	0.0000000	0.0000000	0.0000000
463	0	0	0	0	0	0	0	0	0	0	-2	0	0	2	0.0000000	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
464	0	0	0	0	0	0	0	0	0	0	-2	0	0	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
465	0	0	0	0	0	0	0	0	0	-3	0	0	3	0	0.0000006	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
466	0	0	0	0	0	0	0	0	0	-2	-1	0	2	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
467	0	0	0	0	0	0	0	0	0	-1	0	-2	3	0	-0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
468	0	0	0	0	0	0	0	0	0	-4	0	0	4	0	-0.0000012	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
469	0	0	0	0	0	0	0	0	0	2	1	-2	0	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
470	0	0	0	0	0	0	0	0	0	2	-1	0	-2	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
471	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-0.0000005	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
472	0	0	0	0	0	0	0	0	0	-1	2	0	1	0	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
473	0	0	0	0	0	0	0	0	0	-2	1	2	0	2	-0.0000007	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
474	0	0	0	0	0	0	0	0	0	1	1	0	-1	1	0.0000007	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
475	0	0	0	0	0	0	0	0	0	1	0	1	-2	1	0.0000000	0.0000000	-0.0000012	0.0000000	0.0000000	-0.0000010
476	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
477	0	0	0	0	0	0	0	0	0	1	-1	2	-3	1	0.0000003	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
478	0	0	0	0	0	0	0	0	0	-1	1	2	-1	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
479	0	0	0	0	0	0	0	0	0	-2	0	4	-2	2	-0.0000007	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
480	0	0	0	0	0	0	0	0	0	-2	0	4	-2	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
481	0	0	0	0	0	0	0	0	0	-2	-2	0	2	1	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
482	0	0	0	0	0	0	0	0	0	-2	0	-2	4	0	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
483	0	0	0	0	0	0	0	0	0	1	2	2	-4	1	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
484	0	0	0	0	0	0	0	0	0	1	1	2	-4	2	0.0000007	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
485	0	0	0	0	0	0	0	0	0	-1	2	2	-2	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
486	0	0	0	0	0	0	0	0	0	2	0	0	-3	1	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
487	0	0	0	0	0	0	0	0	0	-1	2	0	0	1	-0.0000005	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
488	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0.0000005	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
489	0	0	0	0	0	0	0	0	0	-1	-1	2	-2	2	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
490	0	0	0	0	0	0	0	0	0	-1	1	0	0	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
491	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	-0.0000008	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
492	0	0	0	0	0	0	0	0	0	-2	1	0	1	0	0.0000009	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
493	0	0	0	0	0	0	0	0	0	1	-2	0	-2	1	0.0000006	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
494	0	0	0	0	0	0	0	0	0	1	0	-2	0	2	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
495	0	0	0	0	0	0	0	0	0	-3	1	0	2	0	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
496	0	0	0	0	0	0	0	0	0	-1	1	-2	2	0	-0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
497	0	0	0	0	0	0	0	0	0	-1	-1	0	0	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
498	0	0	0	0	0	0	0	0	0	-3	0	0	2	0	0.0000005	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
499	0	0	0	0	0	0	0	0	0	-3	-1	0	2	0	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
500	0	0	0	0	0	0	0	0	0	2	0	2	-6	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
501	0	0	0	0	0	0	0	0	0	0	1	2	-4	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
502	0	0	0	0	0	0	0	0	0	2	0	0	-4	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
503	0	0	0	0	0	0	0	0	0	-2	1	2	-2	1	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
504	0	0	0	0	0	0	0	0	0	0	-1	2	-4	1	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
505	0	0	0	0	0	0	0	0	0	0	1	0	-2	2	0.0000009	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
506	0	0	0	0	0	0	0	0	0	-1	0	0	-2	0	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
507	0	0	0	0	0	0	0	0	0	2	0	-2	-2	1	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
508	0	0	0	0	0	0	0	0	0	-4	0	2	0	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
509	0	0	0	0	0	0	0	0	0	-1	-1	0	-1	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
510	0	0	0	0	0	0	0	0	0	0	0	-2	0	2	0.0000009	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
511	0	0	0	0	0	0	0	0	0	-3	0	0	1	0	-0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
512	0	0	0	0	0	0	0	0	0	-1	0	-2	1	0	-0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
513	0	0	0	0	0	0	0	0	0	-2	0	-2	2	1	0.0000003	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
514	0	0	0	0	0	0	0	0	0	0	0	-4	2	0	0.0000008	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
515	0	0	0	0	0	0	0	0	0	-2	-1	-2	2	0	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
516	0	0	0	0	0	0	0	0	0	1	0	2	-6	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
517	0	0	0	0	0	0	0	0	0	-1	0	2	-4	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
518	0	0	0	0	0	0	0	0	0	1	0	0	-4	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
519	0	0	0	0	0	0	0	0	0	2	1	2	-4	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
520	0	0	0	0	0	0	0	0	0	2	1	2	-4	1	0.0000006	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
521	0	0	0	0	0	0	0	0	0	0	1	4	-4	4	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
522	0	0	0	0	0	0	0	0	0	0	1	4	-4	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
523	0	0	0	0	0	0	0	0	0	-1	-1	-2	4	0	-0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
524	0	0	0	0	0	0	0	0	0	-1	-3	0	2	0	0.0000009	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
525	0	0	0	0	0	0	0	0	0	-1	0	-2	4	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
526	0	0	0	0	0	0	0	0	0	-2	-1	0	3	0	-0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
527	0	0	0	0	0	0	0	0	0	0	0	-2	3	0	-0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
528	0	0	0	0	0	0	0	0	0	-2	0	0	3	1	-0.0000005	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
529	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	-0.0000013	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
530	0	0	0	0	0	0	0	0	0	-3	0	2	2	0	-0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
531	0	0	0	0	0	0	0	0	0	1	1	-2	2	0	0.0000010	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
532	0	0	0	0	0	0	0	0	0	-1	1	0	2	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
533	0	0	0	0	0	0	0	0	0	1	-2	2	-2	1	0.0000010	0.0000000	0.0000013	0.0000006	0.0000000	-0.0000005
534	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0.0000000	0.0000000	0.0000030	0.0000000	0.0000000	0.0000014
535	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0.0000000	0.0000000	-0.0000162	0.0000000	0.0000000	-0.0000138
536	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0.0000000	0.0000000	0.0000075	0.0000000	0.0000000	0.0000000
537	0	0	0	0	0	0	0	0	0	-1	2	0	2	1	-0.0000007	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
538	0	0	0	0	0	0	0	0	0	0	0	2	0	2	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
539	0	0	0	0	0	0	0	0	0	-2	0	2	0	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
540	0	0	0	0	0	0	0	0	0	2	0	0	-1	1	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
541	0	0	0	0	0	0	0	0	0	3	0	0	-2	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
542	0	0	0	0	0	0	0	0	0	1	0	2	-2	3	-0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
543	0	0	0	0	0	0	0	0	0	1	2	0	0	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
544	0	0	0	0	0	0	0	0	0	2	0	2	-3	2	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
545	0	0	0	0	0	0	0	0	0	-1	1	4	-2	2	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
546	0	0	0	0	0	0	0	0	0	-2	-2	0	4	0	0.0000006	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
547	0	0	0	0	0	0	0	0	0	0	-3	0	2	0	0.0000009	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
548	0	0	0	0	0	0	0	0	0	0	0	-2	4	0	0.0000005	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
549	0	0	0	0	0	0	0	0	0	-1	-1	0	3	0	-0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
550	0	0	0	0	0	0	0	0	0	-2	0	0	4	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
551	0	0	0	0	0	0	0	0	0	-1	0	0	3	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
552	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
553	0	0	0	0	0	0	0	0	0	1	-1	0	1	0	-0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
554	0	0	0	0	0	0	0	0	0	-1	0	0	2	0	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
555	0	0	0	0	0	0	0	0	0	0	-2	2	0	1	-0.0000006	0.0000000	-0.0000003	0.0000003	0.0000000	0.0000001
556	0	0	0	0	0	0	0	0	0	-1	0	1	2	1	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000	-0.0000002
557	0	0	0	0	0	0	0	0	0	-1	1	0	3	0	0.0000011	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
558	0	0	0	0	0	0	0	0	0	-1	-1	2	1	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
559	0	0	0	0	0	0	0	0	0	0	-1	2	0	0	0.0000011	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
560	0	0	0	0	0	0	0	0	0	-2	1	2	2	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
561	0	0	0	0	0	0	0	0	0	2	-2	2	-2	2	-0.0000001	0.0000000	0.0000003	0.0000003	0.0000000	-0.0000001
562	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
563	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0.0000000	0.0000000	-0.0000013	0.0000000	0.0000000	-0.0000011
564	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0.0000003	0.0000000	0.0000006	0.0000000	0.0000000	0.0000000
565	0	0	0	0	0	0	0	0	0	0	2	0	2	0	-0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
566	0	0	0	0	0	0	0	0	0	2	-1	2	-2	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
567	0	0	0	0	0	0	0	0	0	0	-1	4	-2	1	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
568	0	0	0	0	0	0	0	0	0	0	0	4	-2	3	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
569	0	0	0	0	0	0	0	0	0	0	1	4	-2	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
570	0	0	0	0	0	0	0	0	0	4	0	2	-4	2	-0.0000007	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
571	0	0	0	0	0	0	0	0	0	2	2	2	-2	2	0.0000008	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
572	0	0	0	0	0	0	0	0	0	2	0	4	-4	2	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
573	0	0	0	0	0	0	0	0	0	-1	-2	0	4	0	0.0000011	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
574	0	0	0	0	0	0	0	0	0	-1	-3	2	2	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
575	0	0	0	0	0	0	0	0	0	-3	0	2	4	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
576	0	0	0	0	0	0	0	0	0	-3	0	2	-2	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
577	0	0	0	0	0	0	0	0	0	-1	-1	0	-2	1	0.0000008	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
578	0	0	0	0	0	0	0	0	0	-3	0	0	0	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
579	0	0	0	0	0	0	0	0	0	-3	0	-2	2	0	0.0000011	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
580	0	0	0	0	0	0	0	0	0	0	1	0	-4	1	-0.0000006	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
581	0	0	0	0	0	0	0	0	0	-2	1	0	-2	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
582	0	0	0	0	0	0	0	0	0	-4	0	0	0	1	-0.0000008	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
583	0	0	0	0	0	0	0	0	0	-1	0	0	-4	1	-0.0000007	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
584	0	0	0	0	0	0	0	0	0	-3	0	0	-2	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
585	0	0	0	0	0	0	0	0	0	0	0	0	3	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
586	0	0	0	0	0	0	0	0	0	-1	1	0	4	1	0.0000006	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
587	0	0	0	0	0	0	0	0	0	1	-2	2	0	1	-0.0000006	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
588	0	0	0	0	0	0	0	0	0	0	1	0	3	0	0.0000006	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
589	0	0	0	0	0	0	0	0	0	-1	0	2	2	3	0.0000006	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
590	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
591	0	0	0	0	0	0	0	0	0	-2	0	2	2	2	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
592	0	0	0	0	0	0	0	0	0	-1	1	2	2	0	-0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
593	0	0	0	0	0	0	0	0	0	3	0	0	0	2	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
594	0	0	0	0	0	0	0	0	0	2	1	0	1	0	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
595	0	0	0	0	0	0	0	0	0	2	-1	2	-1	2	0.0000006	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
596	0	0	0	0	0	0	0	0	0	0	0	2	0	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
597	0	0	0	0	0	0	0	0	0	0	0	3	0	3	0.0000000	0.0000000	-0.0000026	0.0000000	0.0000000	-0.0000011
598	0	0	0	0	0	0	0	0	0	0	0	3	0	2	0.0000000	0.0000000	-0.0000010	0.0000000	0.0000000	-0.0000005
599	0	0	0	0	0	0	0	0	0	-1	2	2	2	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
600	0	0	0	0	0	0	0	0	0	-1	0	4	0	0	-0.0000013	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
601	0	0	0	0	0	0	0	0	0	1	2	2	0	1	0.0000003	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
602	0	0	0	0	0	0	0	0	0	3	1	2	-2	1	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
603	0	0	0	0	0	0	0	0	0	1	1	4	-2	2	0.0000007	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
604	0	0	0	0	0	0	0	0	0	-2	-1	0	6	0	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
605	0	0	0	0	0	0	0	0	0	0	-2	0	4	0	0.0000005	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
606	0	0	0	0	0	0	0	0	0	-2	0	0	6	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
607	0	0	0	0	0	0	0	0	0	-2	-2	2	4	2	-0.0000006	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
608	0	0	0	0	0	0	0	0	0	0	-3	2	2	2	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
609	0	0	0	0	0	0	0	0	0	0	0	0	4	2	-0.0000007	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
610	0	0	0	0	0	0	0	0	0	-1	-1	2	3	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
611	0	0	0	0	0	0	0	0	0	-2	0	2	4	0	0.0000013	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
612	0	0	0	0	0	0	0	0	0	2	-1	0	2	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
613	0	0	0	0	0	0	0	0	0	1	0	0	3	0	-0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
614	0	0	0	0	0	0	0	0	0	0	1	0	4	1	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
615	0	0	0	0	0	0	0	0	0	0	1	0	4	0	-0.0000011	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
616	0	0	0	0	0	0	0	0	0	1	-1	2	1	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
617	0	0	0	0	0	0	0	0	0	0	0	2	2	3	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
618	0	0	0	0	0	0	0	0	0	1	0	2	2	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
619	0	0	0	0	0	0	0	0	0	-1	0	2	2	2	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
620	0	0	0	0	0	0	0	0	0	-2	0	4	2	1	0.0000006	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
621	0	0	0	0	0	0	0	0	0	2	1	0	2	1	0.0000003	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
622	0	0	0	0	0	0	0	0	0	2	1	0	2	0	-0.0000012	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
623	0	0	0	0	0	0	0	0	0	2	-1	2	0	0	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
624	0	0	0	0	0	0	0	0	0	1	0	2	1	0	-0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
625	0	0	0	0	0	0	0	0	0	0	1	2	2	0	-0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
626	0	0	0	0	0	0	0	0	0	2	0	2	0	3	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
627	0	0	0	0	0	0	0	0	0	3	0	2	0	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
628	0	0	0	0	0	0	0	0	0	1	0	2	0	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
629	0	0	0	0	0	0	0	0	0	1	0	3	0	3	0.0000000	0.0000000	-0.0000005	0.0000000	0.0000000	-0.0000002
630	0	0	0	0	0	0	0	0	0	1	1	2	1	1	-0.0000007	0.0000000	0.0000000	0.0000004	0.0000000	0.0000000
631	0	0	0	0	0	0	0	0	0	0	2	2	2	2	0.0000006	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
632	0	0	0	0	0	0	0	0	0	2	1	2	0	0	-0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
633	0	0	0	0	0	0	0	0	0	2	0	4	-2	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
634	0	0	0	0	0	0	0	0	0	4	1	2	-2	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
635	0	0	0	0	0	0	0	0	0	-1	-1	0	6	0	0.0000003	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
636	0	0	0	0	0	0	0	0	0	-3	-1	2	6	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
637	0	0	0	0	0	0	0	0	0	-1	0	0	6	1	-0.0000005	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
638	0	0	0	0	0	0	0	0	0	-3	0	2	6	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
639	0	0	0	0	0	0	0	0	0	1	-1	0	4	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
640	0	0	0	0	0	0	0	0	0	1	-1	0	4	0	0.0000012	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
641	0	0	0	0	0	0	0	0	0	-2	0	2	5	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
642	0	0	0	0	0	0	0	0	0	1	-2	2	2	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
643	0	0	0	0	0	0	0	0	0	3	-1	0	2	0	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
644	0	0	0	0	0	0	0	0	0	1	-1	2	2	0	0.0000006	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
645	0	0	0	0	0	0	0	0	0	0	0	2	3	1	0.0000005	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
646	0	0	0	0	0	0	0	0	0	-1	1	2	4	1	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
647	0	0	0	0	0	0	0	0	0	0	1	2	3	2	-0.0000006	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
648	0	0	0	0	0	0	0	0	0	-1	0	4	2	1	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
649	0	0	0	0	0	0	0	0	0	2	0	2	1	1	0.0000006	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
650	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0.0000006	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
651	0	0	0	0	0	0	0	0	0	2	1	2	1	2	-0.0000006	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
652	0	0	0	0	0	0	0	0	0	1	0	4	0	1	0.0000003	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
653	0	0	0	0	0	0	0	0	0	3	1	2	0	1	0.0000007	0.0000000	0.0000000	-0.0000004	0.0000000	0.0000000
654	0	0	0	0	0	0	0	0	0	3	0	4	-2	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
655	0	0	0	0	0	0	0	0	0	-2	-1	2	6	2	-0.0000005	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
656	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0.0000005	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
657	0	0	0	0	0	0	0	0	0	0	-2	2	4	2	-0.0000006	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
658	0	0	0	0	0	0	0	0	0	-2	0	2	6	1	-0.0000006	0.0000000	0.0000000	0.0000003	0.0000000	0.0000000
659	0	0	0	0	0	0	0	0	0	2	0	0	4	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
660	0	0	0	0	0	0	0	0	0	2	0	0	4	0	0.0000010	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
661	0	0	0	0	0	0	0	0	0	2	-2	2	2	2	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
662	0	0	0	0	0	0	0	0	0	0	0	2	4	0	0.0000007	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
663	0	0	0	0	0	0	0	0	0	1	0	2	3	2	0.0000007	0.0000000	0.0000000	-0.0000003	0.0000000	0.0000000
664	0	0	0	0	0	0	0	0	0	4	0	0	2	0	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
665	0	0	0	0	0	0	0	0	0	2	0	2	2	0	0.0000011	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
666	0	0	0	0	0	0	0	0	0	0	0	4	2	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
667	0	0	0	0	0	0	0	0	0	4	-1	2	0	2	-0.0000006	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
668	0	0	0	0	0	0	0	0	0	3	0	2	1	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
669	0	0	0	0	0	0	0	0	0	2	1	2	2	1	0.0000003	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
670	0	0	0	0	0	0	0	0	0	4	1	2	0	2	0.0000005	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
671	0	0	0	0	0	0	0	0	0	-1	-1	2	6	2	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
672	0	0	0	0	0	0	0	0	0	-1	0	2	6	1	-0.0000004	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
673	0	0	0	0	0	0	0	0	0	1	-1	2	4	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
674	0	0	0	0	0	0	0	0	0	1	1	2	4	2	0.0000004	0.0000000	0.0000000	-0.0000002	0.0000000	0.0000000
675	0	0	0	0	0	0	0	0	0	3	1	2	2	2	0.0000003	0.0000000	0.0000000	-0.0000001	0.0000000	0.0000000
676	0	0	0	0	0	0	0	0	0	5	0	2	0	1	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
677	0	0	0	0	0	0	0	0	0	2	-1	2	4	2	-0.0000003	0.0000000	0.0000000	0.0000001	0.0000000	0.0000000
678	0	0	0	0	0	0	0	0	0	2	0	2	4	1	-0.0000003	0.0000000	0.0000000	0.0000002	0.0000000	0.0000000
679	0	0	8	-16	4	5	0	0	0	0	0	0	0	0	0.0001440	0.	0.0000000	0.0000000	0.	0.0000000
680	0	0	-8	16	-4	-5	0	0	2	0	0	0	0	0	0.0000056	0.	-0.0000117	-0.0000040	0.	-0.0000042
681	0	0	8	-16	4	5	0	0	2	0	0	0	0	0	0.0000125	0.	-0.0000043	-0.0000054	0.	0.0000000
682	0	0	0	0	0	0	-1	2	2	0	0	0	0	0	0.0000000	0.	0.0000005	0.0000000	0.	0.0000000
683	0	0	-4	8	-1	-5	0	0	2	0	0	0	0	0	0.0000003	0.	-0.0000007	0.0000000	0.	-0.0000003
684	0	0	4	-8	3	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
685	0	0	3	-8	3	0	0	0	0	0	0	1	-1	1	-0.0000114	0.	0.0000000	0.0000061	0.	0.0000000
686	0	10	-3	0	0	0	0	0	0	-1	0	0	0	0	-0.0000219	0.	0.0000089	0.0000000	0.	0.0000000
687	0	0	0	0	-2	6	-3	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
688	0	0	4	-8	3	0	0	0	0	0	0	0	0	0	-0.0000462	0.	0.0001604	0.0000000	0.	0.0000000
689	0	0	-5	8	-3	0	0	0	0	0	0	1	-1	1	0.0000099	0.	0.0000000	-0.0000053	0.	0.0000000
690	0	0	-4	8	-3	0	0	0	1	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
691	0	0	4	-8	1	5	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000006	0.0000000	0.	0.0000002
692	0	-5	6	4	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
693	0	0	0	0	2	-5	0	0	2	0	0	0	0	0	-0.0000012	0.	0.0000000	0.0000000	0.	0.0000000
694	0	0	0	0	2	-5	0	0	1	0	0	0	0	0	0.0000014	0.	-0.0000218	0.0000008	0.	0.0000117
695	0	0	-1	0	2	-5	0	0	0	0	0	1	-1	1	0.0000031	0.	-0.0000481	-0.0000017	0.	-0.0000257
696	0	0	0	0	2	-5	0	0	0	0	0	0	0	0	-0.0000491	0.	0.0000128	0.0000000	0.	0.0000000
697	0	0	-1	0	-2	5	0	0	0	0	0	1	-1	1	-0.0003084	0.	0.0005123	0.0001647	0.	0.0002735
698	0	0	0	0	-2	5	0	0	1	0	0	0	0	0	-0.0001444	0.	0.0002409	-0.0000771	0.	-0.0001286
699	0	0	0	0	-2	5	0	0	2	0	0	0	0	0	0.0000011	0.	-0.0000024	-0.0000009	0.	-0.0000011
700	0	0	3	-7	0	0	0	0	0	2	0	-1	-1	0	0.0000026	0.	-0.0000009	0.0000000	0.	0.0000000
701	0	19	-21	3	0	0	0	0	0	1	0	0	-2	0	0.0000103	0.	-0.0000060	0.0000000	0.	0.0000000
702	0	2	-4	0	-3	0	0	0	0	0	0	1	-1	1	0.0000000	0.	-0.0000013	0.0000000	0.	-0.0000007
703	0	0	-1	0	2	0	0	0	0	1	0	0	-1	1	-0.0000026	0.	-0.0000029	0.0000014	0.	-0.0000016
704	0	0	-1	0	-4	10	0	0	0	0	0	1	-1	1	0.0000009	0.	-0.0000027	-0.0000005	0.	-0.0000014
705	0	0	2	0	0	-5	0	0	0	-2	0	0	2	1	0.0000012	0.	0.0000000	-0.0000006	0.	0.0000000
706	0	3	-7	4	0	0	0	0	0	0	0	0	0	0	-0.0000007	0.	0.0000000	0.0000000	0.	0.0000000
707	0	0	1	0	1	-1	0	0	0	0	0	-1	1	0	0.0000000	0.	0.0000024	0.0000000	0.	0.0000000
708	0	0	2	0	-2	0	0	0	0	-2	0	0	2	1	0.0000284	0.	0.0000000	-0.0000151	0.	0.0000000
709	0	18	-16	0	0	0	0	0	0	-1	0	0	0	0	0.0000226	0.	0.0000101	0.0000000	0.	0.0000000
710	0	0	1	0	-2	0	0	0	0	-2	0	1	1	2	0.0000000	0.	-0.0000008	0.0000000	0.	-0.0000002
711	0	18	-17	0	0	0	0	0	0	-1	0	1	-1	1	0.0000000	0.	-0.0000006	0.0000000	0.	-0.0000003
712	0	0	2	-2	0	0	0	0	0	-1	0	0	1	1	0.0000005	0.	0.0000000	-0.0000003	0.	0.0000000
713	0	-8	13	0	0	0	0	0	2	0	0	0	0	0	-0.0000041	0.	0.0000175	0.0000017	0.	0.0000076
714	0	-8	11	0	0	0	0	0	0	0	0	2	-2	2	0.0000000	0.	0.0000015	0.0000000	0.	0.0000006
715	0	-8	13	0	0	0	0	0	1	0	0	0	0	0	0.0000425	0.	0.0000212	0.0000269	0.	-0.0000133
716	0	-8	12	0	0	0	0	0	0	0	0	1	-1	1	0.0001200	0.	0.0000598	-0.0000641	0.	0.0000319
717	0	8	-13	0	0	0	0	0	0	0	0	0	0	0	0.0000235	0.	0.0000334	0.0000000	0.	0.0000000
718	0	8	-14	0	0	0	0	0	0	0	0	1	-1	1	0.0000011	0.	-0.0000012	-0.0000006	0.	-0.0000007
719	0	8	-13	0	0	0	0	0	1	0	0	0	0	0	0.0000005	0.	-0.0000006	0.0000003	0.	0.0000003
720	0	0	2	0	-4	5	0	0	0	-2	0	0	2	1	-0.0000005	0.	0.0000000	0.0000003	0.	0.0000000
721	0	3	-3	0	0	0	0	0	0	-2	0	0	2	2	0.0000006	0.	0.0000000	-0.0000003	0.	0.0000000
722	0	0	2	0	-3	1	0	0	0	-2	0	0	2	0	0.0000015	0.	0.0000000	0.0000000	0.	0.0000000
723	0	3	-5	0	2	0	0	0	0	0	0	0	0	1	0.0000013	0.	0.0000000	-0.0000007	0.	0.0000000
724	0	0	2	0	-4	3	0	0	0	-2	0	0	2	0	-0.0000006	0.	-0.0000009	0.0000000	0.	0.0000000
725	0	0	0	2	0	0	0	0	0	0	0	-1	1	0	0.0000266	0.	-0.0000078	0.0000000	0.	0.0000000
726	0	0	-1	2	0	0	0	0	0	0	0	0	0	1	-0.0000460	0.	-0.0000435	0.0000246	0.	-0.0000232
727	0	0	-2	2	0	0	0	0	0	0	0	1	-1	2	0.0000000	0.	0.0000015	0.0000000	0.	0.0000007
728	0	3	-5	0	0	0	0	0	0	-1	0	1	0	1	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
729	0	3	-4	0	0	0	0	0	0	-1	0	0	1	0	0.0000000	0.	0.0000131	0.0000000	0.	0.0000000
730	0	0	2	0	-2	-2	0	0	0	-2	0	0	2	0	0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
731	0	0	-5	9	0	0	0	0	0	-2	0	2	0	2	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
732	0	0	-1	0	0	0	-1	0	0	0	0	1	-1	1	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
733	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
734	0	0	-1	0	0	0	0	2	0	0	0	1	-1	1	-0.0000017	0.	-0.0000019	0.0000009	0.	-0.0000010
735	0	0	0	0	0	0	0	2	1	0	0	0	0	0	-0.0000009	0.	-0.0000011	-0.0000005	0.	0.0000006
736	0	0	0	0	0	0	0	2	2	0	0	0	0	0	-0.0000006	0.	0.0000000	0.0000003	0.	0.0000000
737	0	0	3	-4	0	0	0	0	0	-1	0	0	1	0	-0.0000016	0.	0.0000008	0.0000000	0.	0.0000000
738	0	0	1	0	0	2	0	0	0	0	0	-1	1	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
739	0	0	-1	0	0	2	0	0	0	0	0	1	-1	2	0.0000011	0.	0.0000024	-0.0000005	0.	0.0000011
740	0	0	-9	17	0	0	0	0	0	0	0	0	0	1	-0.0000003	0.	-0.0000004	0.0000001	0.	-0.0000002
741	0	-3	5	0	0	0	0	0	0	0	0	0	0	2	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
742	0	0	-1	0	-1	2	0	0	0	0	0	1	-1	1	0.0000000	0.	-0.0000008	0.0000000	0.	-0.0000004
743	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
744	0	17	-16	0	-2	0	0	0	0	1	0	0	-2	0	0.0000000	0.	0.0000005	0.0000000	0.	0.0000000
745	0	0	-1	0	1	-3	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000002
746	0	0	5	-6	0	0	0	0	0	-2	0	0	2	1	-0.0000006	0.	0.0000004	0.0000003	0.	0.0000002
747	0	0	9	-13	0	0	0	0	0	0	0	-2	2	0	-0.0000003	0.	-0.0000005	0.0000000	0.	0.0000000
748	0	0	-1	0	0	1	0	0	0	0	0	1	-1	2	-0.0000005	0.	0.0000000	0.0000002	0.	0.0000000
749	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0.0000004	0.	0.0000024	-0.0000002	0.	0.0000013

Term	Fundamental Argument Multipliers $M_{i,j}$														Δψ Coefficients			Δε Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
750	0	0	1	0	0	1	0	0	0	0	0	-1	1	0	-0.0000042	0.	0.0000020	0.0000000	0.	0.0000000
751	0	5	-6	0	0	0	0	0	0	0	0	-2	2	0	-0.0000010	0.	0.0000233	0.0000000	0.	0.0000000
752	0	5	-7	0	0	0	0	0	0	0	0	-1	1	1	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
753	0	6	-8	0	0	0	0	0	0	-2	0	0	2	0	0.0000078	0.	-0.0000018	0.0000000	0.	0.0000000
754	0	-6	7	0	0	0	0	0	0	2	0	1	-3	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
755	0	0	0	0	1	0	0	0	0	0	0	0	0	2	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
756	0	0	1	0	1	0	0	0	0	0	0	-1	1	1	0.0000000	0.	-0.0000004	0.0000001	0.	-0.0000002
757	0	0	-1	0	0	0	2	0	0	0	0	1	-1	1	0.0000000	0.	-0.0000008	-0.0000001	0.	-0.0000004
758	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	0.0000003
759	0	0	0	0	0	0	2	0	2	0	0	0	0	0	-0.0000007	0.	0.0000000	0.0000003	0.	0.0000000
760	0	0	-8	15	0	0	0	0	2	0	0	0	0	0	-0.0000014	0.	0.0000008	0.0000006	0.	0.0000003
761	0	0	-8	15	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000008	0.0000000	0.	-0.0000004
762	0	0	-9	15	0	0	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000019	0.0000000	0.	0.0000010
763	0	0	8	-15	0	0	0	0	0	0	0	0	0	0	0.0000045	0.	-0.0000022	0.0000000	0.	0.0000000
764	0	0	8	-15	0	0	0	0	0	1	0	-1	-1	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
765	0	2	-5	0	0	0	0	0	0	2	0	0	-2	0	0.0000000	0.	-0.0000003	0.0000000	0.	0.0000000
766	0	0	2	0	-5	5	0	0	0	-2	0	0	2	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
767	0	0	-6	8	0	0	0	0	0	2	0	0	-2	1	0.0000003	0.	0.0000005	-0.0000002	0.	0.0000003
768	0	0	-2	0	3	0	0	0	0	2	0	0	-2	1	0.0000089	0.	-0.0000016	-0.0000048	0.	-0.0000009
769	0	0	1	0	-3	0	0	0	0	-2	0	1	1	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
770	0	0	1	0	-3	0	0	0	0	-2	0	1	1	1	-0.0000003	0.	0.0000007	0.0000002	0.	0.0000004
771	0	0	2	0	-3	0	0	0	0	-2	0	0	2	0	-0.0000349	0.	-0.0000062	0.0000000	0.	0.0000000
772	0	0	6	-8	0	0	0	0	0	-2	0	0	2	0	-0.0000015	0.	0.0000022	0.0000000	0.	0.0000000
773	0	0	2	0	-1	-5	0	0	0	-2	0	0	2	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
774	0	0	1	0	-1	0	0	0	0	-1	0	0	1	0	-0.0000053	0.	0.0000000	0.0000000	0.	0.0000000
775	0	-20	20	0	0	0	0	0	0	-1	0	1	1	1	0.0000005	0.	0.0000000	-0.0000003	0.	0.0000000
776	0	20	-21	0	0	0	0	0	0	1	0	0	-2	0	0.0000000	0.	-0.0000008	0.0000000	0.	0.0000000
777	0	0	8	-15	0	0	0	0	0	0	0	0	0	1	0.0000015	0.	-0.0000007	-0.0000008	0.	-0.0000004
778	0	0	-10	15	0	0	0	0	0	0	0	2	-2	1	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
779	0	0	1	0	1	0	0	0	0	0	0	-1	1	0	-0.0000021	0.	-0.0000078	0.0000000	0.	0.0000000
780	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0.0000020	0.	-0.0000070	-0.0000011	0.	-0.0000037
781	0	0	-1	0	1	0	0	0	0	0	0	1	-1	2	0.0000000	0.	0.0000006	0.0000000	0.	0.0000003
782	0	0	-1	0	-2	4	0	0	0	0	0	1	-1	1	0.0000005	0.	0.0000003	-0.0000002	0.	0.0000002
783	0	-6	8	0	0	0	0	0	0	2	0	0	-2	1	-0.0000017	0.	-0.0000004	0.0000009	0.	-0.0000002
784	0	5	-6	0	0	0	0	0	0	0	0	-2	2	1	0.0000000	0.	0.0000006	0.0000000	0.	0.0000003
785	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	0.0000032	0.	0.0000015	0.0000017	0.	-0.0000008
786	0	0	-1	0	0	-1	0	0	0	0	0	1	-1	1	0.0000174	0.	0.0000084	-0.0000093	0.	0.0000045
787	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0.0000011	0.	0.0000056	0.0000000	0.	0.0000000
788	0	0	-1	0	0	1	0	0	0	0	0	1	-1	1	-0.0000066	0.	-0.0000012	0.0000035	0.	-0.0000006
789	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0.0000047	0.	0.0000008	-0.0000025	0.	0.0000004
790	0	0	0	0	0	1	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000008	0.0000000	0.	0.0000004
791	0	0	-9	13	0	0	0	0	0	0	0	2	-2	1	0.0000010	0.	-0.0000022	-0.0000005	0.	-0.0000012
792	0	0	7	-13	0	0	0	0	0	0	0	0	0	1	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
793	0	0	5	-6	0	0	0	0	0	-2	0	0	2	0	-0.0000024	0.	0.0000012	0.0000000	0.	0.0000000
794	0	0	9	-17	0	0	0	0	0	0	0	0	0	0	0.0000005	0.	-0.0000006	0.0000000	0.	0.0000000
795	0	0	-9	17	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
796	0	0	-3	4	0	0	0	0	0	1	0	0	-1	1	0.0000004	0.	0.0000003	-0.0000002	0.	0.0000001
797	0	-3	4	0	0	0	0	0	0	1	0	0	-1	1	0.0000000	0.	0.0000029	0.0000000	0.	0.0000015
798	0	0	-1	2	0	0	0	0	0	0	0	0	0	2	-0.0000005	0.	-0.0000004	0.0000002	0.	-0.0000002
799	0	0	0	2	0	0	0	0	0	0	0	-1	1	1	0.0000008	0.	-0.0000003	-0.0000005	0.	-0.0000001
800	1	0	-2	0	0	0	0	0	0	0	0	-2	2	0	0.0000000	0.	-0.0000003	0.0000000	0.	0.0000000
801	0	3	-5	0	2	0	0	0	0	0	0	0	0	0	0.0000010	0.	0.0000000	0.0000000	0.	0.0000000
802	0	0	2	0	-3	1	0	0	0	-2	0	0	2	1	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
803	0	3	-3	0	0	0	0	0	0	-2	0	0	2	1	-0.0000005	0.	0.0000000	0.0000003	0.	0.0000000
804	0	8	-13	0	0	0	0	0	0	0	0	0	0	1	0.0000046	0.	0.0000066	-0.0000025	0.	0.0000035
805	0	8	-12	0	0	0	0	0	0	0	0	-1	1	0	-0.0000014	0.	0.0000007	0.0000000	0.	0.0000000
806	0	-8	11	0	0	0	0	0	0	0	0	2	-2	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000002
807	0	0	2	-2	0	0	0	0	0	-1	0	0	1	0	-0.0000005	0.	0.0000000	0.0000000	0.	0.0000000
808	0	18	-16	0	0	0	0	0	0	-1	0	0	0	1	-0.0000068	0.	-0.0000034	0.0000036	0.	-0.0000018
809	0	0	-1	0	-1	1	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000014	0.0000000	0.	0.0000007
810	0	3	-7	4	0	0	0	0	0	0	0	0	0	1	0.0000010	0.	-0.0000006	-0.0000005	0.	-0.0000003
811	0	0	-3	7	0	0	0	0	0	-2	0	1	1	1	-0.0000005	0.	-0.0000004	0.0000003	0.	-0.0000002
812	0	0	-1	0	-2	5	0	0	0	0	0	1	-1	2	-0.0000003	0.	0.0000005	0.0000001	0.	0.0000002
813	0	0	0	0	-2	5	0	0	0	0	0	0	0	1	0.0000076	0.	0.0000017	-0.0000041	0.	0.0000009
814	0	0	-4	8	-3	0	0	0	0	0	0	0	0	1	0.0000084	0.	0.0000298	-0.0000045	0.	0.0000159
815	0	-10	3	0	0	0	0	0	0	1	0	0	0	1	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
816	0	0	-2	0	0	0	0	0	0	0	0	2	-2	1	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
817	0	10	-3	0	0	0	0	0	0	-1	0	0	0	1	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
818	0	0	4	-8	3	0	0	0	0	0	0	0	0	1	-0.0000082	0.	0.0000292	0.0000044	0.	0.0000156
819	0	0	0	0	2	-5	0	0	0	0	0	0	0	1	-0.0000073	0.	0.0000017	0.0000039	0.	0.0000009
820	0	0	1	0	2	-5	0	0	0	0	0	-1	1	0	-0.0000009	0.	-0.0000016	0.0000000	0.	0.0000000
821	0	0	3	-7	0	0	0	0	0	2	0	-1	-1	1	0.0000003	0.	0.0000000	-0.0000002	0.	-0.0000001
822	0	0	2	0	0	-5	0	0	0	-2	0	0	2	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
823	0	-3	7	-4	0	0	0	0	0	0	0	0	0	1	-0.0000009	0.	-0.0000005	0.0000005	0.	-0.0000003
824	0	0	2	0	-2	0	0	0	0	-2	0	0	2	0	-0.0000439	0.	0.0000000	0.0000000	0.	0.0000000
825	0	-18	16	0	0	0	0	0	0	1	0	0	0	1	0.0000057	0.	-0.0000028	-0.0000030	0.	-0.0000015
826	0	0	1	0	-2	0	0	0	0	-2	0	1	1	1	0.0000000	0.	-0.0000006	0.0000000	0.	-0.0000003
827	0	-8	12	0	0	0	0	0	0	0	0	1	-1	2	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
828	0	-8	13	0	0	0	0	0	0	0	0	0	0	1	-0.0000040	0.	0.0000057	0.0000021	0.	0.0000030
829	0	0	1	-2	0	0	0	0	1	0	0	0	0	0	0.0000023	0.	0.0000007	-0.0000013	0.	0.0000003
830	0	0	0	-2	0	0	0	0	0	0	0	1	-1	1	0.0000273	0.	0.0000080	-0.0000146	0.	0.0000043
831	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	-0.0000449	0.	0.0000430	0.0000000	0.	0.0000000
832	0	0	-2	2	0	0	0	0	0	0	0	1	-1	1	-0.0000008	0.	-0.0000047	0.0000004	0.	-0.0000025
833	0	0	-1	2	0	0	0	0	1	0	0	0	0	0	0.0000006	0.	0.0000047	-0.0000003	0.	0.0000025
834	0	3	-4	0	0	0	0	0	0	-1	0	0	1	1	0.0000000	0.	0.0000023	0.0000000	0.	0.0000013
835	0	0	3	-4	0	0	0	0	0	-1	0	0	1	1	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
836	0	0	-1	0	0	-2	0	0	0	0	0	1	-1	1	0.0000003	0.	-0.0000004	-0.0000002	0.	-0.0000002
837	0	0	-1	0	0	2	0	0	0	0	0	1	-1	1	-0.0000048	0.	-0.0000110	0.0000026	0.	-0.0000059
838	0	0	0	0	0	2	0	0	1	0	0	0	0	0	0.0000051	0.	0.0000114	-0.0000027	0.	0.0000061
839	0	0	0	0	0	2	0	0	2	0	0	0	0	0	-0.0000133	0.	0.0000000	0.0000057	0.	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
840	0	3	-6	0	0	0	0	0	0	0	0	1	-1	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000000
841	0	-3	5	0	0	0	0	0	0	0	0	0	0	1	-0.0000021	0.	-0.0000006	0.0000011	0.	-0.0000003
842	0	-3	4	0	0	0	0	0	0	0	0	1	-1	2	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
843	0	0	-2	4	0	0	0	0	0	0	0	0	0	1	-0.0000011	0.	-0.0000021	0.0000006	0.	-0.0000011
844	0	-5	6	0	0	0	0	0	0	0	0	2	-2	1	-0.0000018	0.	-0.0000436	0.0000009	0.	-0.0000233
845	0	5	-7	0	0	0	0	0	0	0	0	-1	1	0	0.0000035	0.	-0.0000007	0.0000000	0.	0.0000000
846	0	5	-8	0	0	0	0	0	0	0	0	0	0	1	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
847	0	6	-8	0	0	0	0	0	0	-2	0	0	2	1	0.0000011	0.	-0.0000003	-0.0000006	0.	-0.0000001
848	0	0	-8	15	0	0	0	0	0	0	0	0	0	1	-0.0000005	0.	-0.0000003	0.0000003	0.	-0.0000001
849	0	0	2	0	-3	0	0	0	0	-2	0	0	2	1	-0.0000053	0.	-0.0000009	0.0000028	0.	-0.0000005
850	0	0	6	-8	0	0	0	0	0	-2	0	0	2	1	0.0000000	0.	0.0000003	0.0000001	0.	0.0000002
851	0	0	-1	0	1	0	0	0	0	1	0	0	-1	1	0.0000004	0.	0.0000000	-0.0000002	0.	0.0000000
852	0	0	0	0	3	-5	0	0	0	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000000
853	0	0	-1	0	-1	0	0	0	0	0	0	1	-1	1	-0.0000050	0.	0.0000194	0.0000027	0.	0.0000103
854	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	-0.0000013	0.	0.0000052	0.0000007	0.	0.0000028
855	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-0.0000091	0.	0.0000248	0.0000000	0.	0.0000000
856	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0.0000006	0.	0.0000049	-0.0000003	0.	0.0000026
857	0	0	-1	0	1	0	0	0	0	0	0	1	-1	1	-0.0000006	0.	-0.0000047	0.0000003	0.	-0.0000025
858	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
859	0	0	0	0	1	0	0	0	2	0	0	0	0	0	0.0000052	0.	0.0000023	-0.0000023	0.	0.0000010
860	0	0	-1	0	0	-1	0	0	0	0	0	1	-1	2	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
861	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
862	0	0	1	0	0	-1	0	0	0	0	0	-1	1	0	-0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
863	0	0	-7	13	0	0	0	0	2	0	0	0	0	0	-0.0000004	0.	0.0000008	0.0000002	0.	0.0000003
864	0	0	7	-13	0	0	0	0	0	0	0	0	0	0	0.0000010	0.	0.0000000	0.0000000	0.	0.0000000
865	0	0	-5	6	0	0	0	0	0	2	0	0	-2	1	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
866	0	0	-8	11	0	0	0	0	0	0	0	2	-2	1	0.0000000	0.	0.0000008	0.0000000	0.	0.0000004
867	-1	0	2	0	0	0	0	0	0	0	0	2	-2	1	0.0000000	0.	0.0000008	0.0000001	0.	0.0000004
868	0	0	4	-4	0	0	0	0	0	-2	0	0	2	0	-0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
869	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	-0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
870	0	0	-1	0	0	3	0	0	0	0	0	1	-1	1	-0.0000008	0.	0.0000004	0.0000004	0.	0.0000002
871	0	0	0	0	0	3	0	0	1	0	0	0	0	0	0.0000008	0.	-0.0000004	-0.0000004	0.	-0.0000002
872	0	0	0	0	0	3	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000015	0.0000000	0.	0.0000007
873	0	3	-3	0	0	0	0	0	0	-2	0	0	2	0	-0.0000138	0.	0.0000000	0.0000000	0.	0.0000000
874	0	0	-4	8	-3	0	0	0	0	0	0	0	0	2	0.0000000	0.	-0.0000007	0.0000000	0.	-0.0000003
875	0	0	4	-8	3	0	0	0	0	0	0	0	0	2	0.0000000	0.	-0.0000007	0.0000000	0.	-0.0000003
876	0	0	-2	0	2	0	0	0	0	2	0	0	-2	1	0.0000054	0.	0.0000000	-0.0000029	0.	0.0000000
877	0	0	-1	0	2	0	0	0	0	0	0	1	-1	2	0.0000000	0.	0.0000010	0.0000000	0.	0.0000004
878	0	0	0	-2	0	0	0	0	0	0	0	1	-1	2	-0.0000007	0.	0.0000000	0.0000003	0.	0.0000000
879	0	0	1	-2	0	0	0	0	0	0	0	0	0	1	-0.0000037	0.	0.0000035	0.0000020	0.	0.0000019
880	0	0	2	-2	0	0	0	0	0	0	0	-1	1	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000000
881	0	0	1	0	0	-2	0	0	0	0	0	-1	1	0	-0.0000004	0.	0.0000009	0.0000000	0.	0.0000000
882	0	0	-2	0	0	2	0	0	0	0	0	2	-2	1	0.0000008	0.	0.0000000	-0.0000004	0.	0.0000000
883	0	3	-6	0	0	0	0	0	0	0	0	1	-1	1	-0.0000009	0.	-0.0000014	0.0000005	0.	-0.0000008
884	0	3	-5	0	0	0	0	0	1	0	0	0	0	0	-0.0000003	0.	-0.0000009	0.0000003	0.	-0.0000005
885	0	3	-5	0	0	0	0	0	0	0	0	0	0	0	-0.0000145	0.	0.0000047	0.0000000	0.	0.0000000
886	0	-3	4	0	0	0	0	0	0	0	0	1	-1	1	-0.0000010	0.	0.0000040	0.0000005	0.	0.0000021
887	0	-3	5	0	0	0	0	0	1	0	0	0	0	0	0.0000011	0.	-0.0000049	-0.0000007	0.	-0.0000026
888	0	-3	5	0	0	0	0	0	2	0	0	0	0	0	-0.0002150	0.	0.0000000	0.0000932	0.	0.0000000
889	0	-3	3	0	0	0	0	0	0	0	0	2	-2	2	-0.0000012	0.	0.0000000	0.0000005	0.	0.0000000
890	0	-3	5	0	0	0	0	0	2	0	0	0	0	0	0.0000085	0.	0.0000000	-0.0000037	0.	0.0000000
891	0	0	2	-4	0	0	0	0	1	0	0	0	0	0	0.0000004	0.	0.0000000	-0.0000002	0.	0.0000000
892	0	0	1	-4	0	0	0	0	0	0	0	1	-1	1	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
893	0	0	2	-4	0	0	0	0	0	0	0	0	0	0	-0.0000086	0.	0.0000153	0.0000000	0.	0.0000000
894	0	0	-2	4	0	0	0	0	1	0	0	0	0	0	-0.0000006	0.	0.0000009	0.0000003	0.	0.0000005
895	0	0	-3	4	0	0	0	0	0	0	0	1	-1	1	0.0000009	0.	-0.0000013	-0.0000005	0.	-0.0000007
896	0	0	-2	4	0	0	0	0	1	0	0	0	0	0	-0.0000008	0.	0.0000012	0.0000004	0.	0.0000006
897	0	0	-2	4	0	0	0	0	2	0	0	0	0	0	-0.0000051	0.	0.0000000	0.0000022	0.	0.0000000
898	0	-5	8	0	0	0	0	0	2	0	0	0	0	0	-0.0000011	0.	-0.0000268	0.0000005	0.	-0.0000116
899	0	-5	6	0	0	0	0	0	0	0	0	2	-2	2	0.0000000	0.	0.0000012	0.0000000	0.	0.0000005
900	0	-5	8	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000007	0.0000000	0.	0.0000003
901	0	-5	8	0	0	0	0	0	1	0	0	0	0	0	0.0000031	0.	0.0000006	-0.0000017	0.	0.0000003
902	0	-5	7	0	0	0	0	0	0	0	0	1	-1	1	0.0000140	0.	0.0000027	-0.0000075	0.	0.0000014
903	0	-5	8	0	0	0	0	0	1	0	0	0	0	0	0.0000057	0.	0.0000011	-0.0000030	0.	0.0000006
904	0	5	-8	0	0	0	0	0	0	0	0	0	0	0	-0.0000014	0.	-0.0000039	0.0000000	0.	0.0000000
905	0	0	-1	0	-1	0	0	0	0	0	0	1	-1	2	0.0000000	0.	-0.0000006	0.0000000	0.	-0.0000002
906	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	0.0000004	0.	0.0000015	-0.0000002	0.	0.0000008
907	0	0	1	0	-1	0	0	0	0	0	0	-1	1	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000000
908	0	0	-2	0	1	0	0	0	0	0	0	2	-2	1	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
909	0	0	-6	11	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000011	0.0000000	0.	0.0000005
910	0	0	6	-11	0	0	0	0	0	0	0	0	0	0	0.0000009	0.	0.0000006	0.0000000	0.	0.0000000
911	-1	0	4	0	0	0	0	0	2	0	0	0	0	0	-0.0000004	0.	0.0000010	0.0000002	0.	0.0000004
912	1	0	-4	0	0	0	0	0	0	0	0	0	0	0	0.0000005	0.	0.0000003	0.0000000	0.	0.0000000
913	0	-3	3	0	0	0	0	0	0	2	0	0	-2	1	0.0000016	0.	0.0000000	-0.0000009	0.	0.0000000
914	0	0	2	0	0	-2	0	0	0	-2	0	0	2	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
915	0	0	-7	9	0	0	0	0	0	0	0	2	-2	1	0.0000000	0.	0.0000003	-0.0000001	0.	0.0000002
916	0	0	0	0	4	-5	0	0	2	0	0	0	0	0	0.0000007	0.	0.0000000	-0.0000003	0.	0.0000000
917	0	0	0	0	2	0	0	0	0	0	0	0	0	0	-0.0000025	0.	0.0000022	0.0000000	0.	0.0000000
918	0	0	0	0	2	0	0	0	1	0	0	0	0	0	0.0000042	0.	0.0000223	-0.0000022	0.	0.0000119
919	0	0	-1	0	2	0	0	0	0	0	0	1	-1	1	-0.0000027	0.	-0.0000143	0.0000014	0.	-0.0000077
920	0	0	0	0	2	0	0	0	1	0	0	0	0	0	0.0000009	0.	0.0000049	-0.0000005	0.	0.0000026
921	0	0	0	0	2	0	0	0	2	0	0	0	0	0	-0.0001166	0.	0.0000000	0.0000505	0.	0.0000000
922	0	0	-2	0	2	0	0	0	0	0	0	2	-2	2	-0.0000005	0.	0.0000000	0.0000002	0.	0.0000000
923	0	0	0	0	0	5	0	0	2	0	0	0	0	0	-0.0000006	0.	0.0000000	0.0000003	0.	0.0000000
924	0	3	-5	0	0	0	0	0	0	0	0	0	0	1	-0.0000008	0.	0.0000000	0.0000004	0.	0.0000001
925	0	3	-4	0	0	0	0	0	0	0	0	-1	1	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000000
926	0	-3	3	0	0	0	0	0	0	0	0	2	-2	1	0.0000117	0.	0.0000000	-0.0000063	0.	0.0000000
927	0	0	2	-4	0	0	0	0	0	0	0	0	0	1	-0.0000004	0.	0.0000008	0.0000002	0.	0.0000004
928	0	0	-4	4	0	0	0	0	0	0	0	2	-2	1	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
929	0	-5	7	0	0	0	0	0	0	0	0	1	-1	2	-0.0000005	0.	0.0000000	0.0000002	0.	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														Δψ Coefficients			Δε Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
930	0	0	3	-6	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000031	0.0000000	0.	0.0000000
931	0	0	-3	6	0	0	0	0	1	0	0	0	0	0	-0.0000005	0.	0.0000000	0.0000003	0.	0.0000001
932	0	0	-4	6	0	0	0	0	0	0	0	1	-1	1	0.0000004	0.	0.0000000	-0.0000002	0.	0.0000000
933	0	0	-3	6	0	0	0	0	1	0	0	0	0	0	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
934	0	0	-3	6	0	0	0	0	2	0	0	0	0	0	-0.0000024	0.	-0.0000013	0.0000010	0.	-0.0000006
935	0	2	-2	0	0	0	0	0	0	0	0	-1	1	0	0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
936	0	2	-3	0	0	0	0	0	0	0	0	0	0	1	0.0000000	0.	-0.0000032	0.0000000	0.	-0.0000017
937	0	0	-5	9	0	0	0	0	2	0	0	0	0	0	0.0000008	0.	0.0000012	-0.0000003	0.	0.0000005
938	0	0	-5	9	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
939	0	0	5	-9	0	0	0	0	0	0	0	0	0	0	0.0000007	0.	0.0000013	0.0000000	0.	0.0000000
940	0	0	1	0	-2	0	0	0	0	0	0	-1	1	0	-0.0000003	0.	0.0000016	0.0000000	0.	0.0000000
941	0	0	-2	0	2	0	0	0	0	0	0	2	-2	1	0.0000050	0.	0.0000000	-0.0000027	0.	0.0000000
942	0	0	1	0	0	0	0	0	0	-2	0	1	1	1	0.0000000	0.	-0.0000005	0.0000000	0.	-0.0000003
943	0	3	-3	0	0	0	0	0	0	0	0	-2	2	0	0.0000013	0.	0.0000000	0.0000000	0.	0.0000000
944	0	-6	10	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000005	0.0000001	0.	0.0000003
945	0	-6	10	0	0	0	0	0	2	0	0	0	0	0	0.0000024	0.	0.0000005	-0.0000011	0.	0.0000002
946	0	-2	3	0	0	0	0	0	2	0	0	0	0	0	0.0000005	0.	-0.0000011	-0.0000002	0.	-0.0000005
947	0	-2	3	0	0	0	0	0	1	0	0	0	0	0	0.0000030	0.	-0.0000003	-0.0000016	0.	-0.0000002
948	0	-2	2	0	0	0	0	0	0	0	0	1	-1	1	0.0000018	0.	0.0000000	-0.0000009	0.	0.0000000
949	0	2	-3	0	0	0	0	0	0	0	0	0	0	0	0.0000008	0.	0.0000614	0.0000000	0.	0.0000000
950	0	2	-3	0	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	-0.0000003	-0.0000002	0.	-0.0000001
951	0	0	0	0	3	0	0	0	1	0	0	0	0	0	0.0000006	0.	0.0000017	-0.0000003	0.	0.0000009
952	0	0	-1	0	3	0	0	0	0	0	0	1	-1	1	-0.0000003	0.	-0.0000009	0.0000002	0.	-0.0000005
953	0	0	0	0	3	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000006	-0.0000001	0.	0.0000003
954	0	0	0	0	3	0	0	0	2	0	0	0	0	0	-0.0000127	0.	0.0000021	0.0000055	0.	0.0000009
955	0	0	4	-8	0	0	0	0	0	0	0	0	0	0	0.0000003	0.	0.0000005	0.0000000	0.	0.0000000
956	0	0	-4	8	0	0	0	0	2	0	0	0	0	0	-0.0000006	0.	-0.0000010	0.0000003	0.	-0.0000004
957	0	0	2	0	-2	0	0	0	0	0	0	-2	2	0	0.0000005	0.	0.0000000	0.0000000	0.	0.0000000
958	0	0	-4	7	0	0	0	0	2	0	0	0	0	0	0.0000016	0.	0.0000009	-0.0000007	0.	0.0000004
959	0	0	-4	7	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
960	0	0	4	-7	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000022	0.0000000	0.	0.0000000
961	0	-2	3	0	0	0	0	0	0	0	0	0	0	1	0.0000000	0.	0.0000019	0.0000000	0.	0.0000010
962	0	0	-2	0	3	0	0	0	0	0	0	2	-2	1	0.0000007	0.	0.0000000	-0.0000004	0.	0.0000000
963	0	0	-5	10	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	-0.0000002
964	0	-1	2	0	0	0	0	0	0	0	0	0	0	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
965	0	0	0	0	4	0	0	0	2	0	0	0	0	0	-0.0000009	0.	0.0000003	0.0000004	0.	0.0000001
966	0	0	-3	5	0	0	0	0	2	0	0	0	0	0	0.0000017	0.	0.0000000	-0.0000007	0.	0.0000000
967	0	0	-3	5	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000003	-0.0000001	0.	-0.0000002
968	0	0	3	-5	0	0	0	0	0	0	0	0	0	0	-0.0000020	0.	0.0000034	0.0000000	0.	0.0000000
969	0	1	-2	0	0	0	0	0	1	0	0	0	0	0	-0.0000010	0.	0.0000000	0.0000005	0.	0.0000001
970	0	1	-3	0	0	0	0	0	0	0	0	1	-1	1	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
971	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0.0000022	0.	-0.0000087	0.0000000	0.	0.0000000
972	0	-1	2	0	0	0	0	0	1	0	0	0	0	0	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
973	0	-1	2	0	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	-0.0000006	0.0000001	0.	-0.0000002
974	0	-7	11	0	0	0	0	0	2	0	0	0	0	0	-0.0000016	0.	-0.0000003	0.0000007	0.	-0.0000001
975	0	-7	11	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000002
976	0	4	-4	0	0	0	0	0	0	0	0	-2	2	0	0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
977	0	0	2	-3	0	0	0	0	0	0	0	0	0	0	-0.0000068	0.	0.0000039	0.0000000	0.	0.0000000
978	0	-4	4	0	0	0	0	0	0	0	0	2	-2	1	0.0000027	0.	0.0000000	-0.0000014	0.	0.0000000
979	0	4	-5	0	0	0	0	0	0	0	0	-1	1	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000000
980	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	-0.0000025	0.	0.0000000	0.0000000	0.	0.0000000
981	0	-4	7	0	0	0	0	0	1	0	0	0	0	0	-0.0000012	0.	-0.0000003	0.0000006	0.	-0.0000002
982	0	-4	6	0	0	0	0	0	0	0	0	1	-1	1	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
983	0	-4	7	0	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000066	-0.0000001	0.	0.0000029
984	0	-4	6	0	0	0	0	0	2	0	0	0	0	0	0.0000490	0.	0.0000000	-0.0000213	0.	0.0000000
985	0	-4	6	0	0	0	0	0	1	0	0	0	0	0	-0.0000022	0.	0.0000093	0.0000012	0.	0.0000049
986	0	-4	5	0	0	0	0	0	0	0	0	1	-1	1	-0.0000007	0.	0.0000028	0.0000004	0.	0.0000015
987	0	-4	6	0	0	0	0	0	1	0	0	0	0	0	-0.0000003	0.	0.0000013	0.0000002	0.	0.0000007
988	0	4	-6	0	0	0	0	0	0	0	0	0	0	0	-0.0000046	0.	0.0000014	0.0000000	0.	0.0000000
989	0	2	-2	0	0	0	0	0	0	-2	0	0	2	0	-0.0000005	0.	0.0000000	0.0000000	0.	0.0000000
990	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0.0000002	0.	0.0000001	0.0000000	0.	0.0000000
991	0	1	0	0	0	0	0	0	0	0	0	-1	1	0	0.0000000	0.	-0.0000003	0.0000000	0.	0.0000000
992	0	1	-1	0	0	0	0	0	0	0	0	0	0	1	-0.0000028	0.	0.0000000	0.0000015	0.	0.0000000
993	0	0	-1	0	5	0	0	0	2	0	0	0	0	0	0.0000005	0.	0.0000000	-0.0000002	0.	0.0000000
994	0	0	1	-3	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
995	0	0	-1	3	0	0	0	0	2	0	0	0	0	0	-0.0000011	0.	0.0000000	0.0000005	0.	0.0000000
996	0	0	-7	12	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
997	0	-1	1	0	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
998	0	-1	1	0	0	0	0	0	1	0	0	0	0	0	0.0000025	0.	0.0000106	-0.0000013	0.	0.0000057
999	0	-1	0	0	0	0	0	0	0	0	0	1	-1	1	0.0000005	0.	0.0000021	-0.0000003	0.	0.0000011
1000	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0.0001485	0.	0.0000000	0.0000000	0.	0.0000000
1001	0	1	-1	0	0	0	0	0	1	0	0	0	0	0	-0.0000007	0.	-0.0000032	0.0000004	0.	-0.0000017
1002	0	1	-2	0	0	0	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
1003	0	0	-2	5	0	0	0	0	2	0	0	0	0	0	-0.0000006	0.	-0.0000003	0.0000003	0.	-0.0000002
1004	0	0	-1	0	4	0	0	0	2	0	0	0	0	0	0.0000030	0.	-0.0000006	-0.0000013	0.	-0.0000002
1005	0	0	1	0	-4	0	0	0	0	0	0	0	0	0	-0.0000004	0.	0.0000004	0.0000000	0.	0.0000000
1006	0	-1	1	0	0	0	0	0	0	0	0	0	0	1	-0.0000019	0.	0.0000000	0.0000010	0.	0.0000000
1007	0	0	-6	10	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000004	-0.0000001	0.	0.0000002
1008	0	0	-6	10	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
1009	0	0	-3	0	3	0	0	0	0	0	0	2	-2	1	0.0000004	0.	0.0000000	-0.0000002	0.	0.0000000
1010	0	0	-3	7	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
1011	0	4	-4	0	0	0	0	0	0	-2	0	0	2	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1012	0	0	-5	8	0	0	0	0	2	0	0	0	0	0	0.0000005	0.	0.0000003	-0.0000002	0.	0.0000001
1013	0	0	5	-8	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000011	0.0000000	0.	0.0000000
1014	0	0	-1	0	3	0	0	0	2	0	0	0	0	0	0.0000118	0.	0.0000000	-0.0000052	0.	0.0000000
1015	0	0	-1	0	3	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	-0.0000003
1016	0	0	1	0	-3	0	0	0	0	0	0	0	0	0	-0.0000028	0.	0.0000036	0.0000000	0.	0.0000000
1017	0	2	-4	0	0	0	0	0	0	0	0	0	0	0	0.0000005	0.	-0.0000005	0.0000000	0.	0.0000000
1018	0	-2	4	0	0	0	0	0	1	0	0	0	0	0	0.0000014	0.	-0.0000059	-0.0000008	0.	-0.0000031
1019	0	-2	3	0	0	0	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000009	0.0000001	0.	0.0000005

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
1020	0	-2	4	0	0	0	0	0	2	0	0	0	0	0	-0.0000458	0.	0.0000000	0.0000198	0.	0.0000000
1021	0	-6	9	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000045	0.0000000	0.	-0.0000020
1022	0	-6	9	0	0	0	0	0	1	0	0	0	0	0	0.0000009	0.	0.0000000	-0.0000005	0.	0.0000000
1023	0	6	-9	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	0.0000000
1024	0	0	1	0	-2	0	0	0	0	0	0	0	0	1	0.0000000	0.	-0.0000004	-0.0000001	0.	-0.0000002
1025	0	-2	2	0	0	0	0	0	0	0	0	2	-2	1	0.0000011	0.	0.0000000	-0.0000006	0.	0.0000000
1026	0	0	-4	6	0	0	0	0	2	0	0	0	0	0	0.0000006	0.	0.0000000	-0.0000002	0.	0.0000000
1027	0	0	4	-6	0	0	0	0	0	0	0	0	0	0	-0.0000016	0.	0.0000023	0.0000000	0.	0.0000000
1028	0	3	-4	0	0	0	0	0	0	0	0	0	0	1	0.0000000	0.	-0.0000004	0.0000000	0.	-0.0000002
1029	0	0	-1	0	2	0	0	0	2	0	0	0	0	0	-0.0000005	0.	0.0000000	0.0000002	0.	0.0000000
1030	0	0	1	0	-2	0	0	0	0	0	0	0	0	0	-0.0000166	0.	0.0000269	0.0000000	0.	0.0000000
1031	0	0	1	0	-1	0	0	0	0	0	0	0	0	1	0.0000015	0.	0.0000000	-0.0000008	0.	0.0000000
1032	0	-5	9	0	0	0	0	0	2	0	0	0	0	0	0.0000010	0.	0.0000000	-0.0000004	0.	0.0000000
1033	0	0	3	-4	0	0	0	0	0	0	0	0	0	0	-0.0000078	0.	0.0000045	0.0000000	0.	0.0000000
1034	0	-3	4	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	-0.0000002
1035	0	-3	4	0	0	0	0	0	1	0	0	0	0	0	0.0000007	0.	0.0000000	-0.0000004	0.	0.0000000
1036	0	3	-4	0	0	0	0	0	0	0	0	0	0	0	-0.0000005	0.	0.0000328	0.0000000	0.	0.0000000
1037	0	3	-4	0	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
1038	0	0	2	-2	0	0	0	0	0	0	0	0	0	1	0.0000005	0.	0.0000000	-0.0000002	0.	0.0000000
1039	0	0	-1	0	2	0	0	0	0	0	0	0	0	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1040	0	0	1	0	0	-3	0	0	0	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1041	0	0	1	0	1	-5	0	0	0	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1042	0	0	-1	0	1	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	-0.0000002
1043	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	-0.0001223	0.	-0.0000026	0.0000000	0.	0.0000000
1044	0	0	1	0	-1	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000007	0.0000000	0.	0.0000003
1045	0	0	1	0	-3	5	0	0	0	0	0	0	0	0	0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1046	0	-3	4	0	0	0	0	0	0	0	0	0	0	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000002
1047	0	0	1	0	0	-2	0	0	0	0	0	0	0	0	-0.0000006	0.	0.0000020	0.0000000	0.	0.0000000
1048	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	-0.0000368	0.	0.0000000	0.0000000	0.	0.0000000
1049	0	0	1	0	0	-1	0	0	0	0	0	0	0	0	-0.0000075	0.	0.0000000	0.0000000	0.	0.0000000
1050	0	0	-1	0	1	0	0	0	0	0	0	0	0	1	0.0000011	0.	0.0000000	-0.0000006	0.	0.0000000
1051	0	0	-2	2	0	0	0	0	0	0	0	0	0	1	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
1052	0	-8	14	0	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1053	0	0	1	0	2	-5	0	0	0	0	0	0	0	0	-0.0000013	0.	-0.0000030	0.0000000	0.	0.0000000
1054	0	0	5	-8	3	0	0	0	0	0	0	0	0	0	0.0000021	0.	0.0000003	0.0000000	0.	0.0000000
1055	0	0	5	-8	3	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1056	0	0	-1	0	0	0	0	0	1	0	0	0	0	0	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
1057	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0.0000008	0.	-0.0000027	0.0000000	0.	0.0000000
1058	0	0	3	-8	3	0	0	0	0	0	0	0	0	0	-0.0000019	0.	-0.0000011	0.0000000	0.	0.0000000
1059	0	0	-3	8	-3	0	0	0	2	0	0	0	0	0	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
1060	0	0	1	0	-2	5	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000005	0.0000000	0.	0.0000002
1061	0	-8	12	0	0	0	0	0	2	0	0	0	0	0	-0.0000006	0.	0.0000000	0.0000002	0.	0.0000000
1062	0	-8	12	0	0	0	0	0	0	0	0	0	0	0	-0.0000008	0.	0.0000000	0.0000000	0.	0.0000000
1063	0	0	1	0	1	-2	0	0	0	0	0	0	0	0	-0.0000001	0.	0.0000000	0.0000000	0.	0.0000000
1064	0	0	1	0	0	1	0	0	2	0	0	0	0	0	-0.0000014	0.	0.0000000	0.0000006	0.	0.0000000
1065	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0.0000006	0.	0.0000000	0.0000000	0.	0.0000000
1066	0	0	0	2	0	0	0	0	2	0	0	0	0	0	-0.0000074	0.	0.0000000	0.0000032	0.	0.0000000
1067	0	0	1	0	0	2	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
1068	0	-5	5	0	0	0	0	0	0	0	0	2	-2	1	0.0000004	0.	0.0000000	-0.0000002	0.	0.0000000
1069	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0.0000008	0.	0.0000011	0.0000000	0.	0.0000000
1070	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000002
1071	0	0	1	0	1	0	0	0	2	0	0	0	0	0	-0.0000262	0.	0.0000000	0.0000114	0.	0.0000000
1072	0	3	-6	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000000
1073	0	-3	6	0	0	0	0	0	1	0	0	0	0	0	-0.0000007	0.	0.0000000	0.0000004	0.	0.0000000
1074	0	-3	6	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000027	0.0000000	0.	-0.0000012
1075	0	0	-1	4	0	0	0	0	2	0	0	0	0	0	-0.0000019	0.	-0.0000008	0.0000008	0.	-0.0000004
1076	0	-5	7	0	0	0	0	0	2	0	0	0	0	0	0.0000202	0.	0.0000000	-0.0000087	0.	0.0000000
1077	0	-5	7	0	0	0	0	0	1	0	0	0	0	0	-0.0000008	0.	0.0000035	0.0000005	0.	0.0000019
1078	0	-5	6	0	0	0	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
1079	0	5	-7	0	0	0	0	0	0	0	0	0	0	0	0.0000016	0.	-0.0000005	0.0000000	0.	0.0000000
1080	0	0	-1	0	1	0	0	0	0	0	0	2	-2	1	0.0000005	0.	0.0000000	-0.0000003	0.	0.0000000
1081	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	0.0000000
1082	-1	0	3	0	0	0	0	0	2	0	0	0	0	0	0.0000001	0.	0.0000000	0.0000000	0.	0.0000000
1083	0	0	1	0	2	0	0	0	2	0	0	0	0	0	-0.0000035	0.	-0.0000048	0.0000015	0.	-0.0000021
1084	0	0	-2	6	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	-0.0000005	0.0000001	0.	-0.0000002
1085	0	2	-2	0	0	0	0	0	0	0	0	0	0	1	0.0000006	0.	0.0000000	-0.0000003	0.	0.0000000
1086	0	0	-6	9	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1087	0	0	6	-9	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	0.0000000
1088	0	-2	2	0	0	0	0	0	1	0	0	0	0	0	0.0000012	0.	0.0000055	-0.0000006	0.	0.0000029
1089	0	-2	1	0	0	0	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
1090	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	-0.0000598	0.	0.0000000	0.0000000	0.	0.0000000
1091	0	2	-2	0	0	0	0	0	1	0	0	0	0	0	-0.0000003	0.	-0.0000013	0.0000001	0.	-0.0000007
1092	0	0	1	0	3	0	0	0	2	0	0	0	0	0	-0.0000005	0.	-0.0000007	0.0000002	0.	-0.0000003
1093	0	0	-5	7	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1094	0	0	5	-7	0	0	0	0	0	0	0	0	0	0	0.0000005	0.	-0.0000007	0.0000000	0.	0.0000000
1095	0	-2	2	0	0	0	0	0	0	0	0	0	0	1	0.0000004	0.	0.0000000	-0.0000002	0.	0.0000000
1096	0	0	4	-5	0	0	0	0	0	0	0	0	0	0	0.0000016	0.	-0.0000006	0.0000000	0.	0.0000000
1097	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0.0000008	0.	-0.0000003	0.0000000	0.	0.0000000
1098	0	-1	3	0	0	0	0	0	1	0	0	0	0	0	0.0000008	0.	-0.0000031	-0.0000004	0.	-0.0000016
1099	0	-1	2	0	0	0	0	0	0	0	0	1	-1	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1100	0	-1	3	0	0	0	0	0	2	0	0	0	0	0	0.0000113	0.	0.0000000	-0.0000049	0.	0.0000000
1101	0	-7	10	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000024	0.0000000	0.	-0.0000010
1102	0	-7	10	0	0	0	0	0	1	0	0	0	0	0	0.0000004	0.	0.0000000	-0.0000002	0.	0.0000000
1103	0	0	3	-3	0	0	0	0	0	0	0	0	0	0	0.0000027	0.	0.0000000	0.0000000	0.	0.0000000
1104	0	-4	8	0	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1105	0	-4	5	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	-0.0000002
1106	0	-4	5	0	0	0	0	0	1	0	0	0	0	0	0.0000005	0.	0.0000000	-0.0000002	0.	0.0000000
1107	0	4	-5	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	0.0000000
1108	0	0	1	1	0	0	0	0	2	0	0	0	0	0	-0.0000013	0.	0.0000000	0.0000006	0.	0.0000000
1109	0	0	-2	0	5	0	0	0	2	0	0	0	0	0	0.0000005	0.	0.0000000	-0.0000002	0.	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=1$	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
1110	0	0	0	3	0	0	0	0	2	0	0	0	0	0	-0.0000018	0.	-0.0000010	0.0000008	0.	-0.0000004
1111	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-0.0000004	0.	-0.0000028	0.0000000	0.	0.0000000
1112	0	1	0	0	0	0	0	0	2	0	0	0	0	0	-0.0000005	0.	0.0000006	0.0000002	0.	0.0000003
1113	0	-9	13	0	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1114	0	0	-1	5	0	0	0	0	2	0	0	0	0	0	-0.0000005	0.	-0.0000009	0.0000002	0.	-0.0000004
1115	0	0	-2	0	4	0	0	0	2	0	0	0	0	0	0.0000017	0.	0.0000000	-0.0000007	0.	0.0000000
1116	0	0	2	0	-4	0	0	0	0	0	0	0	0	0	0.0000011	0.	0.0000004	0.0000000	0.	0.0000000
1117	0	0	-2	7	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000006	0.0000000	0.	-0.0000002
1118	0	0	2	0	-3	0	0	0	0	0	0	0	0	0	0.0000083	0.	0.0000015	0.0000000	0.	0.0000000
1119	0	-2	5	0	0	0	0	0	1	0	0	0	0	0	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
1120	0	-2	5	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000114	0.0000000	0.	-0.0000049
1121	0	-6	8	0	0	0	0	0	2	0	0	0	0	0	0.0000117	0.	0.0000000	-0.0000051	0.	0.0000000
1122	0	-6	8	0	0	0	0	0	1	0	0	0	0	0	-0.0000005	0.	0.0000019	0.0000002	0.	0.0000010
1123	0	6	-8	0	0	0	0	0	0	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1124	0	0	2	0	-2	0	0	0	0	0	0	0	0	1	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
1125	0	0	-3	9	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
1126	0	0	5	-6	0	0	0	0	0	0	0	0	0	0	0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1127	0	0	5	-6	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000006	0.0000000	0.	-0.0000002
1128	0	0	2	0	-2	0	0	0	0	0	0	0	0	0	0.0000393	0.	0.0000003	0.0000000	0.	0.0000000
1129	0	0	2	0	-2	0	0	0	1	0	0	0	0	0	-0.0000004	0.	0.0000021	0.0000002	0.	0.0000011
1130	0	0	2	0	-2	0	0	0	2	0	0	0	0	0	-0.0000006	0.	0.0000000	0.0000003	0.	-0.0000001
1131	0	-5	10	0	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000008	0.0000001	0.	0.0000004
1132	0	0	4	-4	0	0	0	0	0	0	0	0	0	0	0.0000008	0.	0.0000000	0.0000000	0.	0.0000000
1133	0	0	4	-4	0	0	0	0	2	0	0	0	0	0	0.0000018	0.	-0.0000029	-0.0000008	0.	-0.0000013
1134	0	-3	3	0	0	0	0	0	1	0	0	0	0	0	0.0000008	0.	0.0000034	-0.0000004	0.	0.0000018
1135	0	3	-3	0	0	0	0	0	0	0	0	0	0	0	0.0000089	0.	0.0000000	0.0000000	0.	0.0000000
1136	0	3	-3	0	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000012	-0.0000001	0.	0.0000006
1137	0	3	-3	0	0	0	0	0	2	0	0	0	0	0	0.0000054	0.	-0.0000015	-0.0000024	0.	-0.0000007
1138	0	0	2	0	0	-3	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
1139	0	0	-5	13	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1140	0	0	2	0	-1	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000035	0.0000000	0.	0.0000000
1141	0	0	2	0	-1	0	0	0	2	0	0	0	0	0	-0.0000154	0.	-0.0000030	0.0000067	0.	-0.0000013
1142	0	0	2	0	0	-2	0	0	0	0	0	0	0	0	0.0000015	0.	0.0000000	0.0000000	0.	0.0000000
1143	0	0	2	0	0	-2	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
1144	0	0	3	-2	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000009	0.0000000	0.	0.0000000
1145	0	0	3	-2	0	0	0	0	2	0	0	0	0	0	0.0000080	0.	-0.0000071	-0.0000035	0.	-0.0000031
1146	0	0	2	0	0	-1	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000020	0.0000000	0.	-0.0000009
1147	0	0	-6	15	0	0	0	0	2	0	0	0	0	0	0.0000011	0.	0.0000005	-0.0000005	0.	0.0000002
1148	0	-8	15	0	0	0	0	0	2	0	0	0	0	0	0.0000061	0.	-0.0000096	-0.0000027	0.	-0.0000042
1149	0	-3	9	-4	0	0	0	0	2	0	0	0	0	0	0.0000014	0.	0.0000009	-0.0000006	0.	0.0000004
1150	0	0	2	0	2	-5	0	0	2	0	0	0	0	0	-0.0000011	0.	-0.0000006	0.0000005	0.	-0.0000003
1151	0	0	-2	8	-1	-5	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
1152	0	0	6	-8	3	0	0	0	2	0	0	0	0	0	0.0000123	0.	-0.0000415	-0.0000053	0.	-0.0000180
1153	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000000	-0.0000035	0.	0.0000000
1154	0	0	2	0	0	0	0	0	0	0	0	0	0	0	-0.0000005	0.	0.0000000	0.0000000	0.	0.0000000
1155	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0.0000007	0.	-0.0000032	-0.0000004	0.	-0.0000017
1156	0	0	1	0	0	0	0	0	0	0	0	1	-1	1	0.0000000	0.	-0.0000009	0.0000000	0.	-0.0000005
1157	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000002
1158	0	0	2	0	0	0	0	0	2	0	0	0	0	0	-0.0000089	0.	0.0000000	0.0000038	0.	0.0000000
1159	0	0	-6	16	-4	-5	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000086	-0.0000006	0.	-0.0000019
1160	0	0	-2	8	-3	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000000	0.0000006	0.	-0.0000019
1161	0	0	-2	8	-3	0	0	0	2	0	0	0	0	0	-0.0000123	0.	-0.0000416	0.0000053	0.	-0.0000180
1162	0	0	6	-8	1	5	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
1163	0	0	2	0	-2	5	0	0	2	0	0	0	0	0	0.0000012	0.	-0.0000006	-0.0000005	0.	-0.0000003
1164	0	3	-5	4	0	0	0	0	2	0	0	0	0	0	-0.0000013	0.	0.0000009	0.0000006	0.	0.0000004
1165	0	-8	11	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000015	0.0000000	0.	-0.0000007
1166	0	-8	11	0	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1167	0	-8	11	0	0	0	0	0	2	0	0	0	0	0	-0.0000062	0.	-0.0000097	0.0000027	0.	-0.0000042
1168	0	0	11	0	0	0	0	0	2	0	0	0	0	0	-0.0000011	0.	0.0000005	0.0000005	0.	0.0000002
1169	0	0	2	0	0	1	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000019	0.0000000	0.	-0.0000008
1170	0	3	-3	0	2	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1171	0	0	4	-8	3	0	0	0	0	0	0	2	-2	1	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
1172	0	0	1	0	0	0	0	0	0	0	0	1	-1	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
1173	0	0	-4	8	-3	0	0	0	0	0	0	2	-2	1	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
1174	0	0	1	2	0	0	0	0	2	0	0	0	0	0	-0.0000085	0.	-0.0000070	0.0000037	0.	-0.0000031
1175	0	0	2	0	1	0	0	0	2	0	0	0	0	0	0.0000163	0.	-0.0000012	-0.0000072	0.	-0.0000005
1176	0	-3	7	0	0	0	0	0	2	0	0	0	0	0	-0.0000063	0.	-0.0000016	0.0000028	0.	-0.0000007
1177	0	0	0	4	0	0	0	0	2	0	0	0	0	0	-0.0000021	0.	-0.0000032	0.0000009	0.	-0.0000014
1178	0	-5	6	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000001
1179	0	-5	6	0	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
1180	0	5	-6	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000008	0.0000000	0.	0.0000000
1181	0	5	-6	0	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000010	-0.0000001	0.	0.0000004
1182	0	0	2	0	2	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1183	0	0	-1	6	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000007	0.0000000	0.	-0.0000003
1184	0	0	7	-9	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	-0.0000002
1185	0	2	-1	0	0	0	0	0	0	0	0	0	0	0	0.0000006	0.	0.0000019	0.0000000	0.	0.0000000
1186	0	2	-1	0	0	0	0	0	2	0	0	0	0	0	0.0000005	0.	-0.0000173	-0.0000002	0.	-0.0000075
1187	0	0	6	-7	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000007	0.0000000	0.	-0.0000003
1188	0	0	5	-5	0	0	0	0	2	0	0	0	0	0	0.0000007	0.	-0.0000012	-0.0000003	0.	-0.0000005
1189	0	-1	4	0	0	0	0	0	1	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
1190	0	-1	4	0	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	-0.0000004	-0.0000001	0.	-0.0000002
1191	0	-7	9	0	0	0	0	0	2	0	0	0	0	0	0.0000074	0.	0.0000000	-0.0000032	0.	0.0000000
1192	0	-7	9	0	0	0	0	0	1	0	0	0	0	0	-0.0000003	0.	0.0000012	0.0000002	0.	0.0000006
1193	0	0	4	-3	0	0	0	0	2	0	0	0	0	0	0.0000026	0.	-0.0000014	-0.0000011	0.	-0.0000006
1194	0	0	3	-1	0	0	0	0	2	0	0	0	0	0	0.0000019	0.	0.0000000	-0.0000008	0.	0.0000000
1195	0	-4	4	0	0	0	0	0	1	0	0	0	0	0	0.0000006	0.	0.0000024	-0.0000003	0.	0.0000013
1196	0	4	-4	0	0	0	0	0	0	0	0	0	0	0	0.0000083	0.	0.0000000	0.0000000	0.	0.0000000
1197	0	4	-4	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000010	0.0000000	0.	-0.0000005
1198	0	4	-4	0	0	0	0	0	2	0	0	0	0	0	0.0000011	0.	-0.0000003	-0.0000005	0.	-0.0000001
1199	0	0	2	1	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000001

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
1200	0	0	-3	0	5	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1201	0	1	1	0	0	0	0	0	0	0	0	0	0	0	-0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
1202	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0.0000005	0.	-0.0000023	-0.0000003	0.	-0.0000012
1203	0	1	1	0	0	0	0	0	2	0	0	0	0	0	-0.0000339	0.	0.0000000	0.0000147	0.	0.0000000
1204	0	-9	12	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000010	0.0000000	0.	-0.0000005
1205	0	0	3	0	-4	0	0	0	0	0	0	0	0	0	0.0000005	0.	0.0000000	0.0000000	0.	0.0000000
1206	0	1	-1	0	0	0	0	0	0	0	0	2	-2	1	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1207	0	0	7	-8	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	-0.0000002
1208	0	0	3	0	-3	0	0	0	0	0	0	0	0	0	0.0000018	0.	-0.0000003	0.0000000	0.	0.0000000
1209	0	0	3	0	-3	0	0	0	2	0	0	0	0	0	0.0000009	0.	-0.0000011	-0.0000004	0.	-0.0000005
1210	0	-2	6	0	0	0	0	0	2	0	0	0	0	0	-0.0000008	0.	0.0000000	0.0000004	0.	0.0000000
1211	0	-6	7	0	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1212	0	6	-7	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000009	0.0000000	0.	0.0000000
1213	0	0	6	-6	0	0	0	0	2	0	0	0	0	0	0.0000006	0.	-0.0000009	-0.0000002	0.	-0.0000004
1214	0	0	3	0	-2	0	0	0	0	0	0	0	0	0	-0.0000004	0.	-0.0000012	0.0000000	0.	0.0000000
1215	0	0	3	0	-2	0	0	0	2	0	0	0	0	0	0.0000067	0.	-0.0000091	-0.0000029	0.	-0.0000039
1216	0	0	5	-4	0	0	0	0	2	0	0	0	0	0	0.0000030	0.	-0.0000018	-0.0000013	0.	-0.0000008
1217	0	3	-2	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000000	0.0000000	0.	0.0000000
1218	0	3	-2	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000114	0.0000000	0.	-0.0000050
1219	0	0	3	0	-1	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000000	0.0000023	0.	0.0000000
1220	0	0	3	0	-1	0	0	0	2	0	0	0	0	0	0.0000517	0.	0.0000016	-0.0000224	0.	0.0000007
1221	0	0	3	0	0	-2	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000007	0.0000000	0.	-0.0000003
1222	0	0	4	-2	0	0	0	0	2	0	0	0	0	0	0.0000143	0.	-0.0000003	-0.0000062	0.	-0.0000001
1223	0	0	3	0	0	-1	0	0	2	0	0	0	0	0	0.0000029	0.	0.0000000	-0.0000013	0.	0.0000000
1224	0	0	1	0	-1	0	0	0	0	0	0	2	-2	1	-0.0000004	0.	0.0000000	0.0000002	0.	0.0000000
1225	0	-8	16	0	0	0	0	0	2	0	0	0	0	0	-0.0000006	0.	0.0000000	0.0000003	0.	0.0000000
1226	0	0	3	0	2	-5	0	0	2	0	0	0	0	0	0.0000005	0.	0.0000012	-0.0000002	0.	0.0000005
1227	0	0	7	-8	3	0	0	0	2	0	0	0	0	0	-0.0000025	0.	0.0000000	0.0000011	0.	0.0000000
1228	0	0	-5	16	-4	-5	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1229	0	0	3	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
1230	0	0	-1	8	-3	0	0	0	2	0	0	0	0	0	-0.0000022	0.	0.0000012	0.0000010	0.	0.0000005
1231	0	-8	10	0	0	0	0	0	2	0	0	0	0	0	0.0000050	0.	0.0000000	-0.0000022	0.	0.0000000
1232	0	-8	10	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000007	0.0000000	0.	0.0000004
1233	0	-8	10	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1234	0	0	2	2	0	0	0	0	2	0	0	0	0	0	-0.0000004	0.	0.0000004	0.0000002	0.	0.0000002
1235	0	0	3	0	1	0	0	0	2	0	0	0	0	0	-0.0000005	0.	-0.0000011	0.0000002	0.	-0.0000005
1236	0	-3	8	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
1237	0	-5	5	0	0	0	0	0	1	0	0	0	0	0	0.0000004	0.	0.0000017	-0.0000002	0.	0.0000009
1238	0	5	-5	0	0	0	0	0	0	0	0	0	0	0	0.0000059	0.	0.0000000	0.0000000	0.	0.0000000
1239	0	5	-5	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000004	0.0000000	0.	-0.0000002
1240	0	5	-5	0	0	0	0	0	2	0	0	0	0	0	-0.0000008	0.	0.0000000	0.0000004	0.	0.0000000
1241	0	2	0	0	0	0	0	0	0	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1242	0	2	0	0	0	0	0	0	1	0	0	0	0	0	0.0000004	0.	-0.0000015	-0.0000002	0.	-0.0000008
1243	0	2	0	0	0	0	0	0	2	0	0	0	0	0	0.0000370	0.	-0.0000008	-0.0000160	0.	0.0000000
1244	0	0	7	-7	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000000	0.0000000	0.	-0.0000003
1245	0	0	7	-7	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1246	0	0	6	-5	0	0	0	0	2	0	0	0	0	0	-0.0000006	0.	0.0000003	0.0000003	0.	0.0000001
1247	0	7	-8	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000006	0.0000000	0.	0.0000000
1248	0	0	5	-3	0	0	0	0	2	0	0	0	0	0	-0.0000010	0.	0.0000000	0.0000004	0.	0.0000000
1249	0	4	-3	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000009	0.0000000	0.	0.0000004
1250	0	1	2	0	0	0	0	0	2	0	0	0	0	0	0.0000004	0.	0.0000017	-0.0000002	0.	0.0000007
1251	0	-9	11	0	0	0	0	0	2	0	0	0	0	0	0.0000034	0.	0.0000000	-0.0000015	0.	0.0000000
1252	0	-9	11	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
1253	0	0	4	0	-4	0	0	0	2	0	0	0	0	0	-0.0000005	0.	0.0000000	0.0000002	0.	0.0000000
1254	0	0	4	0	-3	0	0	0	2	0	0	0	0	0	-0.0000037	0.	-0.0000007	0.0000016	0.	-0.0000003
1255	0	-6	6	0	0	0	0	0	1	0	0	0	0	0	0.0000003	0.	0.0000013	-0.0000002	0.	0.0000007
1256	0	6	-6	0	0	0	0	0	0	0	0	0	0	0	0.0000040	0.	0.0000000	0.0000000	0.	0.0000000
1257	0	6	-6	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000002
1258	0	0	4	0	-2	0	0	0	2	0	0	0	0	0	-0.0000184	0.	-0.0000003	0.0000080	0.	-0.0000001
1259	0	0	6	-4	0	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1260	0	3	-1	0	0	0	0	0	0	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1261	0	3	-1	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000010	-0.0000001	0.	-0.0000006
1262	0	3	-1	0	0	0	0	0	2	0	0	0	0	0	0.0000031	0.	-0.0000006	-0.0000013	0.	0.0000000
1263	0	0	4	0	-1	0	0	0	2	0	0	0	0	0	-0.0000003	0.	-0.0000032	0.0000001	0.	-0.0000014
1264	0	0	4	0	0	-2	0	0	2	0	0	0	0	0	-0.0000007	0.	0.0000000	0.0000003	0.	0.0000000
1265	0	0	5	-2	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	-0.0000008	0.0000000	0.	-0.0000004
1266	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0.0000003	0.	-0.0000004	0.0000000	0.	0.0000000
1267	0	8	-9	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000000
1268	0	5	-4	0	0	0	0	0	2	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1269	0	2	1	0	0	0	0	0	2	0	0	0	0	0	0.0000019	0.	-0.0000023	0.0000002	0.	-0.0000010
1270	0	2	1	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000000	-0.0000010	0.	0.0000000
1271	0	2	1	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000002
1272	0	-7	7	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000009	-0.0000001	0.	0.0000005
1273	0	7	-7	0	0	0	0	0	0	0	0	0	0	0	0.0000028	0.	0.0000000	0.0000000	0.	0.0000000
1274	0	4	-2	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000007	0.0000000	0.	-0.0000004
1275	0	4	-2	0	0	0	0	0	2	0	0	0	0	0	0.0000008	0.	-0.0000004	-0.0000004	0.	0.0000000
1276	0	4	-2	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000000	0.0000000	0.	-0.0000002
1277	0	4	-2	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
1278	0	0	5	0	-4	0	0	0	2	0	0	0	0	0	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1279	0	0	5	0	-3	0	0	0	2	0	0	0	0	0	-0.0000009	0.	0.0000000	0.0000004	0.	0.0000001
1280	0	0	5	0	-2	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000012	-0.0000001	0.	0.0000005
1281	0	3	0	0	0	0	0	0	2	0	0	0	0	0	0.0000017	0.	-0.0000003	0.0000000	0.	-0.0000001
1282	0	-8	8	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000007	0.0000000	0.	0.0000004
1283	0	8	-8	0	0	0	0	0	0	0	0	0	0	0	0.0000019	0.	0.0000000	0.0000000	0.	0.0000000
1284	0	5	-3	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	-0.0000003
1285	0	5	-3	0	0	0	0	0	2	0	0	0	0	0	0.0000014	0.	-0.0000003	-0.0000001	0.	0.0000000
1286	0	-9	9	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000000	0.0000000	0.	-0.0000001
1287	0	-9	9	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000000	-0.0000005	0.	0.0000000
1288	0	-9	9	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
1289	0	9	-9	0	0	0	0	0	0	0	0	0	0	0	0.0000013	0.	0.0000000	0.0000000	0.	0.0000000

Term	Fundamental Argument Multipliers $M_{i,j}$														$\Delta\psi$ Coefficients			$\Delta\epsilon$ Coefficients		
i	$j=$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	S_i	$\dot{S}_i$	C'_i	C_i	$\dot{C}_i$	S'_i
															″	″	″	″	″	″
1290	0	6	-4	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000002
1291	0	0	6	0	0	0	0	0	2	0	0	0	0	0	0.0000002	0.	0.0000009	0.0000003	0.	0.0000004
1292	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0.0000000	0.	0.0000000	-0.0000004	0.	0.0000000
1293	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0.0000008	0.	0.0000000	0.0000000	0.	0.0000000
1294	0	0	6	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000004	0.0000000	0.	0.0000002
1295	0	0	6	0	0	0	0	0	2	0	0	0	0	0	0.0000006	0.	0.0000000	-0.0000003	0.	0.0000000
1296	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0.0000006	0.	0.0000000	0.0000000	0.	0.0000000
1297	0	0	6	0	0	0	0	0	1	0	0	0	0	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1298	0	0	6	0	0	0	0	0	2	0	0	0	0	0	0.0000005	0.	0.0000000	-0.0000002	0.	0.0000000
1299	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1300	0	0	2	0	-2	0	0	0	0	1	0	0	-2	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1301	0	2	-2	0	0	0	0	0	0	1	0	0	-2	0	0.0000006	0.	0.0000000	0.0000000	0.	0.0000000
1302	0	0	1	0	-1	0	0	0	0	1	0	0	-2	0	0.0000007	0.	0.0000000	0.0000000	0.	0.0000000
1303	0	1	-1	0	0	0	0	0	0	1	0	0	-2	0	-0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
1304	0	3	-3	0	0	0	0	0	0	-1	0	0	0	0	0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
1305	0	0	2	0	-2	0	0	0	0	-1	0	0	0	0	0.0000006	0.	0.0000000	0.0000000	0.	0.0000000
1306	0	0	4	-8	3	0	0	0	0	-1	0	0	2	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000000
1307	0	0	4	-8	3	0	0	0	0	1	0	0	-2	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000000
1308	0	0	4	-8	3	0	0	0	0	-2	0	0	2	0	0.0000005	0.	0.0000000	0.0000000	0.	0.0000000
1309	0	0	2	0	-3	0	0	0	0	-1	0	0	0	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1310	0	0	1	0	-1	0	0	0	0	-1	0	0	0	0	0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
1311	0	1	-1	0	0	0	0	0	0	-1	0	0	0	0	-0.0000005	0.	0.0000000	0.0000000	0.	0.0000000
1312	0	2	-2	0	0	0	0	0	0	-1	0	0	2	0	0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
1313	0	0	1	0	0	0	0	0	0	1	0	-1	1	0	0.0000000	0.	0.0000003	0.0000000	0.	0.0000000
1314	0	0	2	0	-3	0	0	0	0	-1	0	0	2	0	0.0000013	0.	0.0000000	0.0000000	0.	0.0000000
1315	0	0	2	0	-3	0	0	0	0	-2	0	0	0	0	0.0000021	0.	0.0000011	0.0000000	0.	0.0000000
1316	0	0	4	-8	3	0	0	0	0	1	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	0.0000000
1317	0	0	-1	0	0	0	0	0	0	-1	0	1	-1	1	0.0000000	0.	-0.0000005	0.0000000	0.	-0.0000002
1318	0	0	-1	0	0	0	0	0	0	1	0	1	-1	1	0.0000000	0.	0.0000005	0.0000000	0.	0.0000003
1319	0	0	4	-8	3	0	0	0	0	-1	0	0	0	0	0.0000000	0.	-0.0000005	0.0000000	0.	0.0000000
1320	0	0	2	0	-2	0	0	0	0	-1	0	0	2	1	-0.0000003	0.	0.0000000	0.0000002	0.	0.0000000
1321	0	0	2	0	-2	0	0	0	0	0	0	0	0	0	0.0000020	0.	0.0000010	0.0000000	0.	0.0000000
1322	0	0	2	0	-2	0	0	0	0	-1	0	0	2	0	-0.0000034	0.	0.0000000	0.0000000	0.	0.0000000
1323	0	3	-3	0	0	0	0	0	0	-1	0	0	2	0	-0.0000019	0.	0.0000000	0.0000000	0.	0.0000000
1324	0	0	-2	0	2	0	0	0	0	1	0	0	-2	1	0.0000003	0.	0.0000000	-0.0000002	0.	0.0000000
1325	0	-3	3	0	0	0	0	0	0	1	0	2	-2	2	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1326	0	0	-2	0	2	0	0	0	0	1	0	2	-2	2	-0.0000006	0.	0.0000000	0.0000003	0.	0.0000000
1327	0	1	-1	0	0	0	0	0	0	1	0	0	0	0	-0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
1328	0	0	1	0	-1	0	0	0	0	1	0	0	0	0	0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1329	0	2	-2	0	0	0	0	0	0	0	0	0	-2	0	0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1330	0	0	1	0	-1	0	0	0	0	0	0	0	-2	0	0.0000004	0.	0.0000000	0.0000000	0.	0.0000000
1331	0	-2	2	0	0	0	0	0	0	0	0	2	0	2	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1332	0	0	-1	0	1	0	0	0	0	0	0	2	0	2	0.0000006	0.	0.0000000	-0.0000003	0.	0.0000000
1333	0	-1	1	0	0	0	0	0	0	0	0	2	0	2	-0.0000008	0.	0.0000000	0.0000003	0.	0.0000000
1334	0	-2	3	0	0	0	0	0	0	0	0	2	0	2	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1335	0	0	2	0	-2	0	0	0	0	0	0	0	2	0	-0.0000003	0.	0.0000000	0.0000000	0.	0.0000000
1336	0	0	1	0	0	0	0	0	0	0	0	1	1	2	0.0000000	0.	-0.0000003	0.0000000	0.	-0.0000002
1337	0	0	1	0	0	0	0	0	0	1	0	2	0	2	0.0000126	0.	-0.0000063	-0.0000055	0.	-0.0000027
1338	0	10	-3	0	0	0	0	0	0	-1	0	2	0	2	-0.0000005	0.	0.0000000	0.0000002	0.	0.0000001
1339	0	0	1	0	0	0	0	0	0	0	0	1	1	1	-0.0000003	0.	0.0000028	0.0000002	0.	0.0000015
1340	0	0	1	0	0	0	0	0	0	1	0	2	0	2	0.0000005	0.	0.0000000	-0.0000002	0.	0.0000001
1341	0	0	4	-8	3	0	0	0	0	0	0	2	0	2	0.0000000	0.	0.0000009	0.0000001	0.	0.0000004
1342	0	0	-4	8	-3	0	0	0	0	0	0	2	0	2	0.0000000	0.	0.0000009	-0.0000001	0.	0.0000004
1343	0	0	-4	8	-3	0	0	0	0	-1	0	2	0	2	-0.0000126	0.	-0.0000063	0.0000055	0.	-0.0000027
1344	0	0	-2	0	3	0	0	0	0	2	0	2	-2	2	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1345	0	0	-2	0	3	0	0	0	0	1	0	2	0	1	0.0000021	0.	-0.0000011	-0.0000011	0.	-0.0000006
1346	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0.0000000	0.	-0.0000004	0.0000000	0.	0.0000000
1347	0	0	1	0	0	0	0	0	0	-1	0	2	0	1	-0.0000021	0.	-0.0000011	0.0000011	0.	-0.0000006
1348	0	0	2	0	-2	0	0	0	0	-2	0	2	2	2	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1349	0	2	-3	0	0	0	0	0	0	0	0	2	0	2	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1350	0	1	-1	0	0	0	0	0	0	0	0	2	0	2	0.0000008	0.	0.0000000	-0.0000004	0.	0.0000000
1351	0	0	1	0	-1	0	0	0	0	0	0	2	0	2	-0.0000006	0.	0.0000000	0.0000003	0.	0.0000000
1352	0	2	-2	0	0	0	0	0	0	0	0	2	0	2	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1353	0	0	-1	0	1	0	0	0	0	-1	0	2	2	2	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1354	0	-1	1	0	0	0	0	0	0	1	0	2	0	2	-0.0000003	0.	0.0000000	0.0000001	0.	0.0000000
1355	0	0	2	0	-3	0	0	0	0	-1	0	2	2	2	-0.0000005	0.	0.0000000	0.0000002	0.	0.0000000
1356	0	0	2	0	-3	0	0	0	0	2	0	2	0	2	0.0000024	0.	-0.0000012	-0.0000011	0.	-0.0000005
1357	0	0	-4	8	-3	0	0	0	0	1	0	2	0	2	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1358	0	0	4	-8	3	0	0	0	0	1	0	2	0	2	0.0000000	0.	0.0000003	0.0000000	0.	0.0000001
1359	0	0	1	0	0	0	0	0	0	1	0	1	1	1	0.0000000	0.	0.0000003	0.0000000	0.	0.0000002
1360	0	0	1	0	0	0	0	0	0	0	0	2	0	2	-0.0000024	0.	-0.0000012	0.0000010	0.	-0.0000005
1361	0	0	1	0	0	0	0	0	0	2	0	2	0	1	0.0000004	0.	0.0000000	-0.0000002	0.	-0.0000001
1362	0	0	2	0	-2	0	0	0	0	-1	0	2	2	2	0.0000013	0.	0.0000000	-0.0000006	0.	0.0000000
1363	0	3	-3	0	0	0	0	0	0	-1	0	2	2	2	0.0000007	0.	0.0000000	-0.0000003	0.	0.0000000
1364	0	1	-1	0	0	0	0	0	0	1	0	2	0	2	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000
1365	0	0	2	0	-2	0	0	0	0	0	0	2	2	2	0.0000003	0.	0.0000000	-0.0000001	0.	0.0000000

Errata & Updates

Errata in this circular and updates to it are given at URL 1. Please consult the web page at this URL periodically.